EE60032: Analog Signal Processing

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Module-2: Filters

Introduction to filters

* Process signals in frequency domain.
* Oldest technology : using L and C. Works well for high frequency.
  For low frequency, L and C will be bulky, introduce parasitics.
* Inductor less filter technology :- Passive RC filter, Active RC filter, switched capacitor filter.
* Classifications of filters :-

Filter Circuits

Based on frequency band

1) Low pass filter
2) High pass filter
3) Band pass filter
4) Band reject/notch filter
5) All pass filter

Based on implementation

1) Continuous time
2) Discrete time

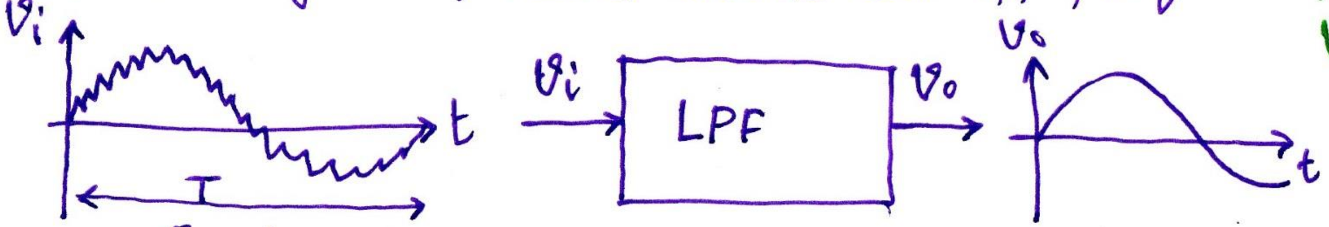
Based on circuit element

1) Passive filter
2) Active filter.

Filter classifications

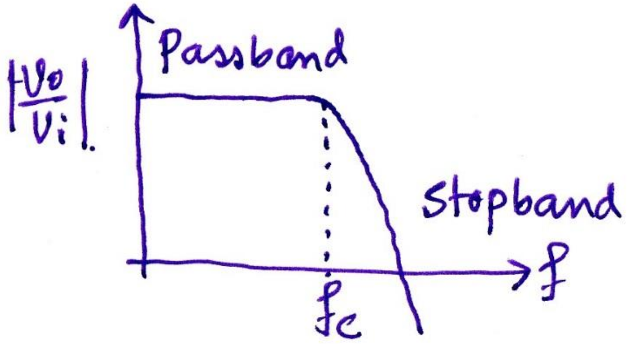
1) Low pass filter :- (LPF)

Passes low frequency components of a signal below cut-off frequency. Blocks the high frequency components above cut-off freq.



$f = \frac{1}{T}$

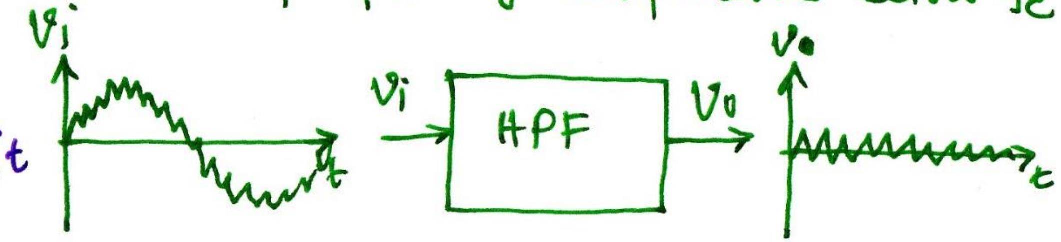
$f < f_c$.

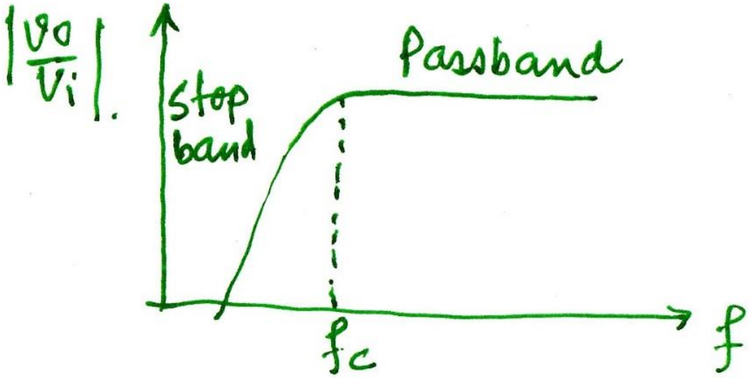


High frequency noise can be filtered out by using LPF.

2) High pass filter : (HPF)

Passes the frequency components above the cut-off frequency. Attenuates the lower frequency components below f_c

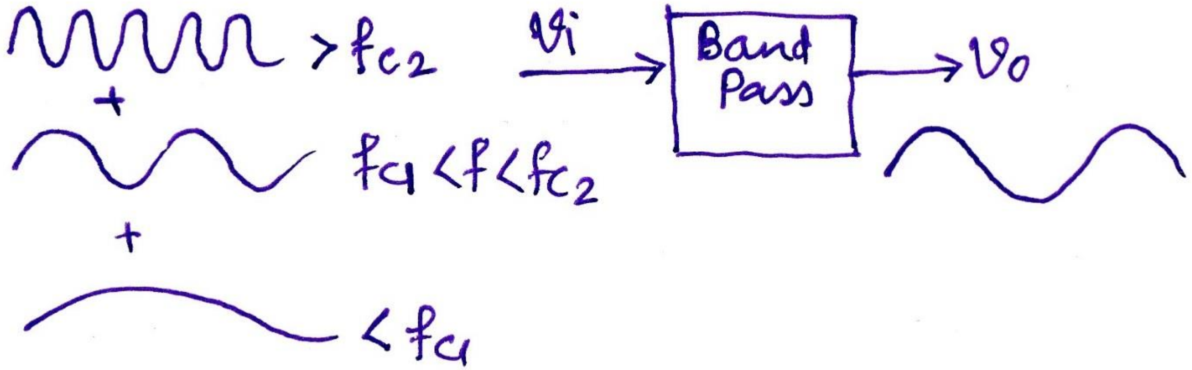


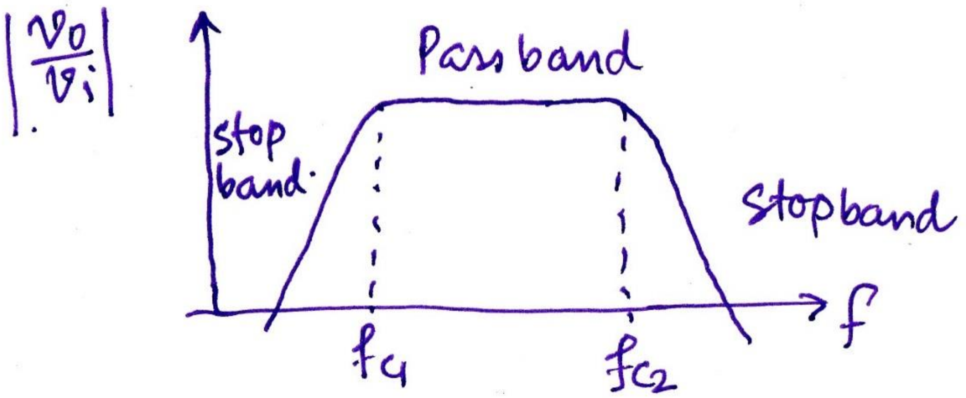


Any rectified voltage has power supply noise of 50/60 Hz. That noise can be filtered out by HPF.

3) Band pass filter :-

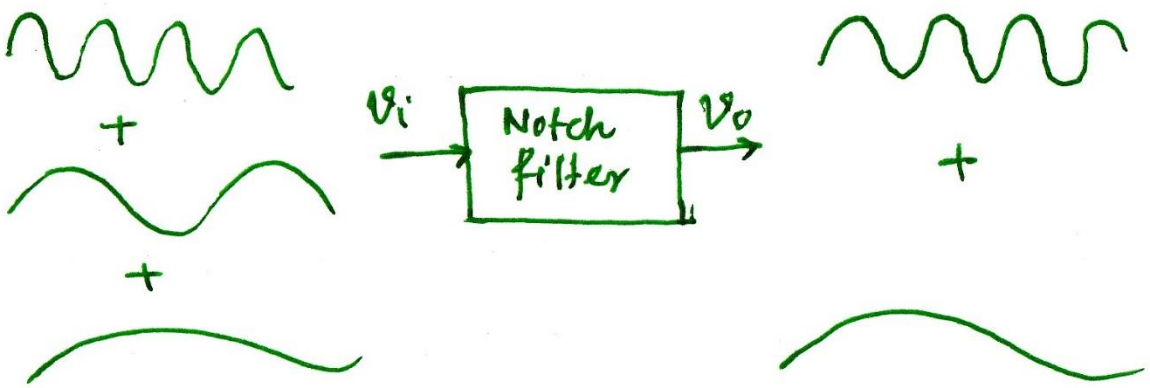
It has two cutoff frequencies. f_{c1} and f_{c2}.
$f_{c1} \leq f \leq f_{c2}$ will be passed only.

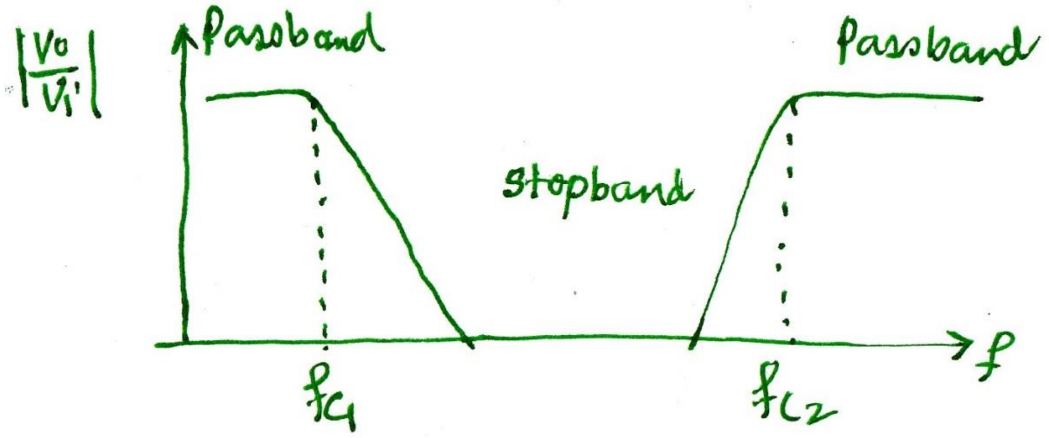




4) Band reject filter / notch filter :-

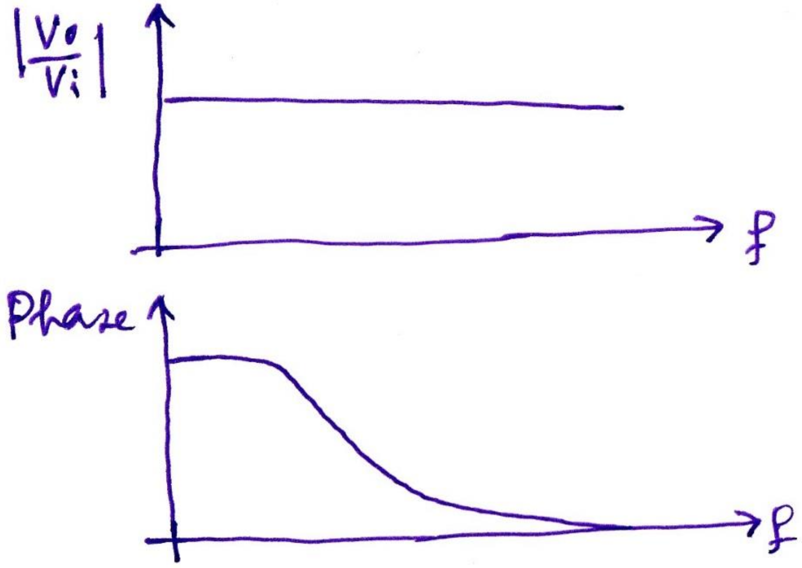
It also has two cut-off frequencies. f_{c1} & f_{c2}.
Here all frequency components passed except $f_{c1} < f < f_{c2}$



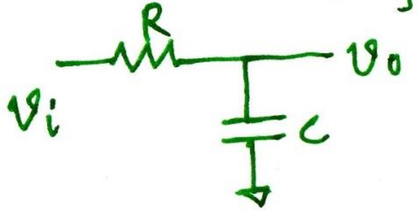


5) All-pass filter / delay filter :-

passes all frequency components with equal gain. Introduce delay based on input frequency.

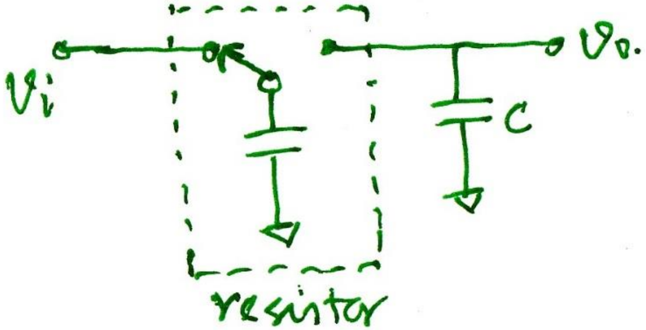


6) Continuous time filter :-



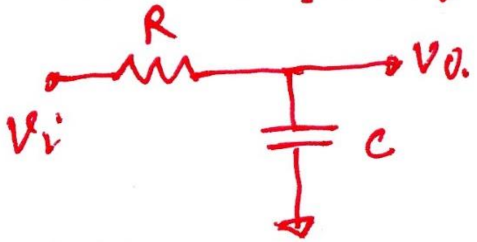
Area consuming

7) Discrete time filter :-

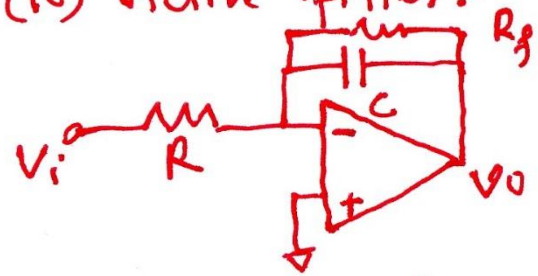


More area efficient.

9) Passive filter :-



(10) Active filter :-



Active implementation provides more design flexibility in filter design.

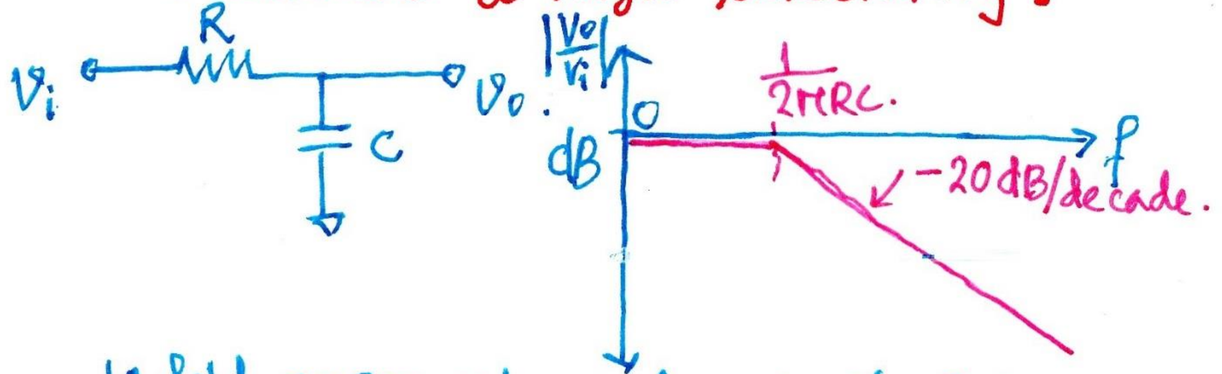
Generalised filter transfer function :-

Basic objective is to achieve a sharp transition from passband to stopband (f selectivity)

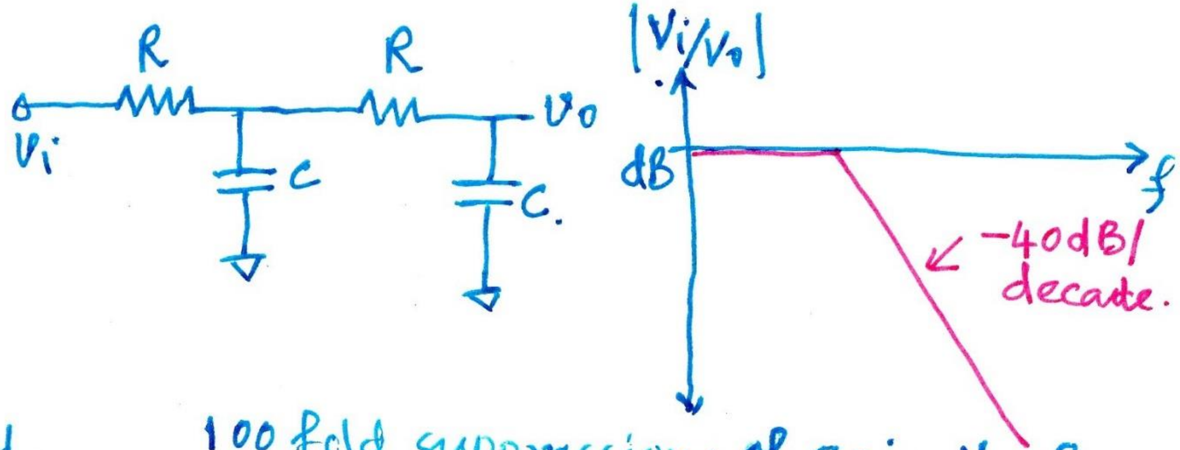
This is because : (1) Interferer frequency may be close to the desired signal band.

(2) Interfering level may be higher than the desired signal level.

How to achieve a high selectivity :-



10 fold suppression of gain if f is increased by 10X.



100 fold suppression of gain if f is increased by 10X.

Increasing the 'order' of the transfer function can improve the frequency selectivity.

The generalised transfer function of a nth order filter :-

$$H(s) = \frac{a_M s^M + a_{M-1} s^{M-1} + \cdots + a_0}{b_N s^N + b_{N-1} s^{N-1} + \cdots + b_0} = \alpha \frac{(s+z_1)(s+z_2)\cdots\cdots(s+z_M)}{(s+p_1)(s+p_2)\cdots\cdots(s+p_N)}$$

where z_K and p_K (real or complex) denote zeros and poles respectively.

z_K or $p_K = \sigma + j\omega$, σ = real part, $j\omega$ = imaginary part.

Few points :-

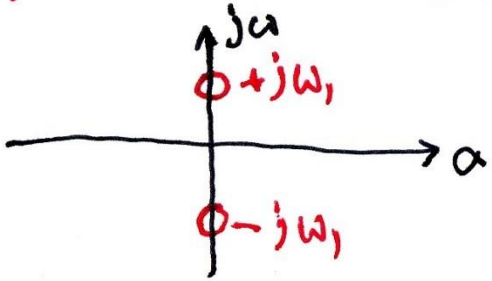
1) n is the order of the filter.

2) $n \geq m$, otherwise if $s \to \infty$, $H(\infty) \to \infty$, not a stable system.

3) Complex pole and zeros must occur in conjugate pair for better optimization.

$p_1 = \sigma_1 + j\omega_1$ and $p_2 = \sigma_1 - j\omega_1$

4) If zeros are located on $j\omega$ axis in s-plane, then $z_{1,2} = \pm j\omega_1$, then the numerator will be $(s - j\omega_1)(s + j\omega_1) = s^2 + \omega_1^2$



At $s = j\omega_1$, $|H(s)|$ drops to zero.

Imaginary zeros are placed at stopband.

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Realization of first order filter :-

$$H(s) = \frac{a_1 s + a_0}{s + \omega_0} = \frac{a_1\left(s + \frac{a_0}{a_1}\right)}{s+\omega_0} \; ; \; \text{pole} = -\omega_0, \quad \text{zero} \cdot z_1 = -\frac{a_0}{a_1}.$$

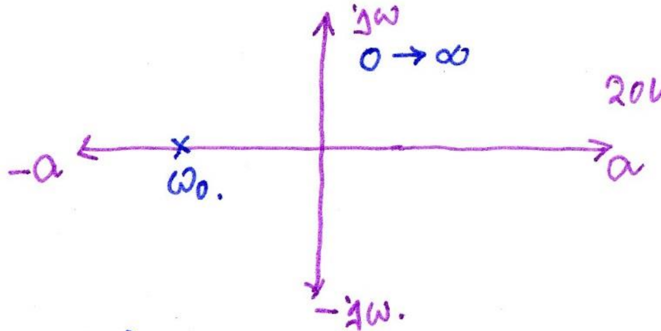
Depending on the location of poles and zero, we get different transfer fn.

Low pass filter : (LPF)

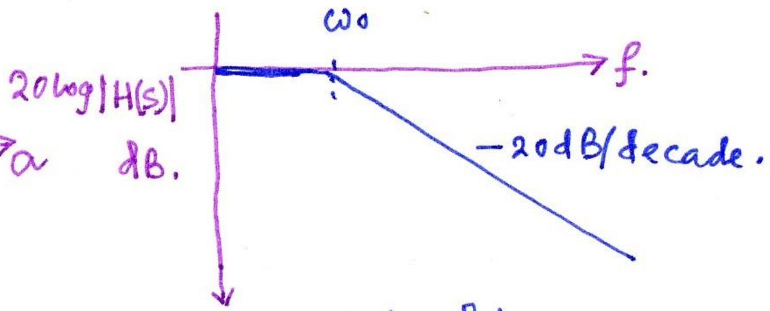
zero occurs at a very high frequency compared to pole frequency. i.e. $z_1 >> p_1$.

$$H(s) = \frac{a_1\left(s+\frac{a_0}{a_1}\right)}{s+\omega_0} \approx \frac{a_1 \; a_0/a_1}{s+\omega_0} \quad \text{for } \frac{a_0}{a_1} >> s$$

$$= \frac{a_0}{s+\omega_0}. \qquad |H(0)| = \frac{a_0}{\omega_0}, \text{ and } p_1 = -\omega_0.$$

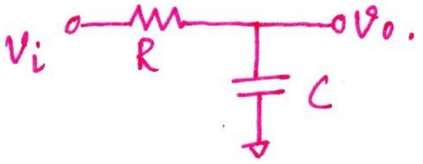


Pole-zero locations at s-plane.



Gain plot.

Passive realization :-

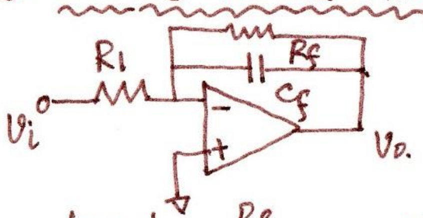


$$H(s) = \frac{1}{1+sRC} = \frac{1}{RC\left(s+\frac{1}{RC}\right)}$$

$$|H(0)| = 1.$$

$$\omega_0 = \frac{1}{RC}.$$

Active realization :-

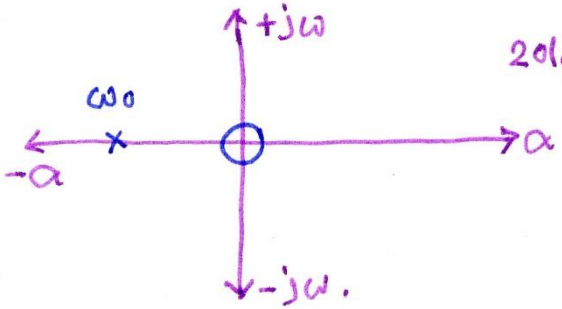


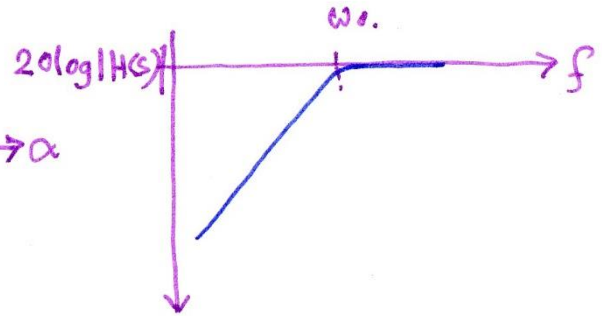
$$H(s) = -\frac{R_f}{R_1} \cdot \frac{1}{1+sC_fR_f}$$

$$|H(0)| = \frac{R_f}{R_1}, \quad \omega_0 = \frac{1}{R_fC_f}$$

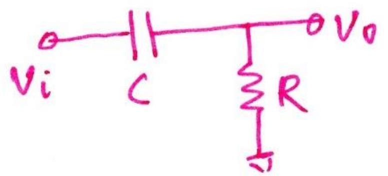
High pass filter : (HPF)

$$H(s) = \frac{a_1\left(s+\frac{a_0}{a_1}\right)}{s+\omega_0} \approx \frac{s a_1\left(1+\frac{a_0}{a_1 s}\right)}{s+\omega_0} \approx \frac{a_1 s}{s+\omega_0} \quad \text{for } s \gg \frac{a_0}{a_1} \; ; \; p_1 = -\omega_0, \; z_1 = 0.$$





Passive realization :-



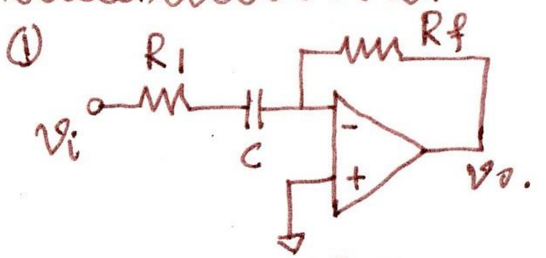
$$H(s) = \frac{sCR}{1+sCR}.$$

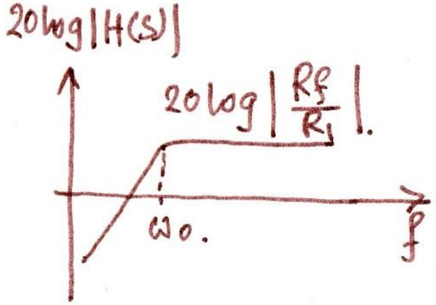
High frequency gain = 1

$$\omega_0 = \frac{1}{RC}.$$

Active realization :-

①



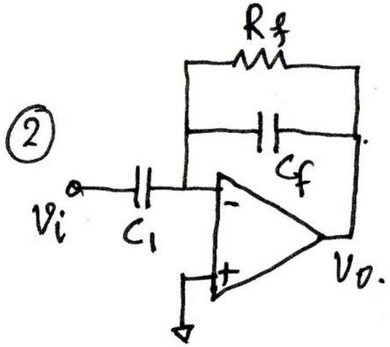


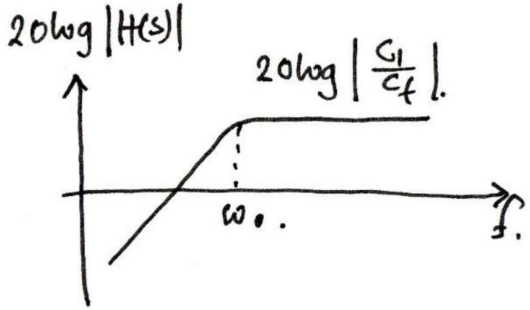
$$H(s) = -\frac{sR_fC}{1+sR_1C}.$$

High frequency gain $= \frac{R_f}{R_1}.$

$$\omega_0 = \frac{1}{R_1C}.$$

②





$$H(s) = -\frac{sC_1R_f}{1+sC_fR_f}$$

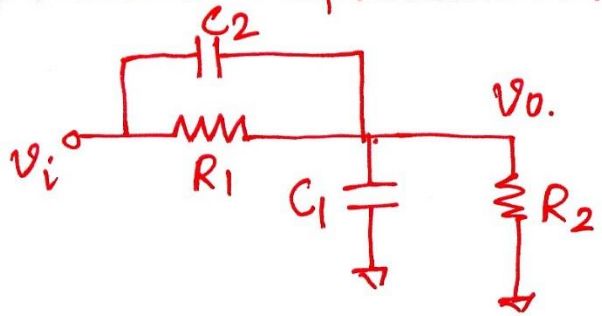
High frequency gain $= \frac{C_1}{C_f}$

$$\omega_0 = \frac{1}{R_fC_f}$$

Note : Band pass and band reject filters can not be realized in first order.

Try yourself :- Generalised structure of low-pass and high-pass filter :

1) Passive implementation :-

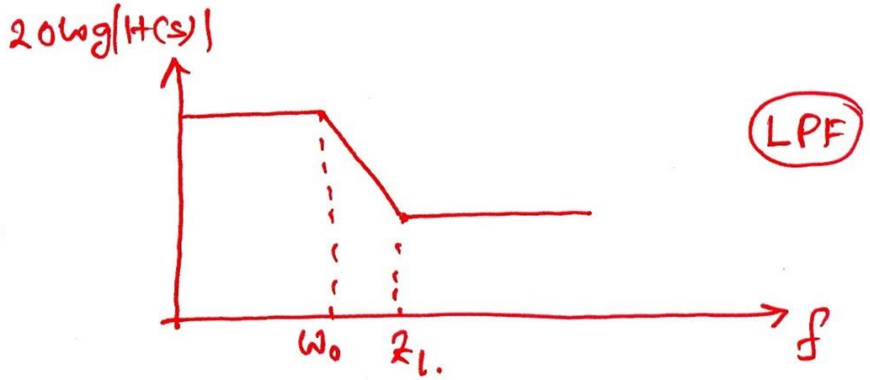


DC gain = ?

High frequency gain = ?

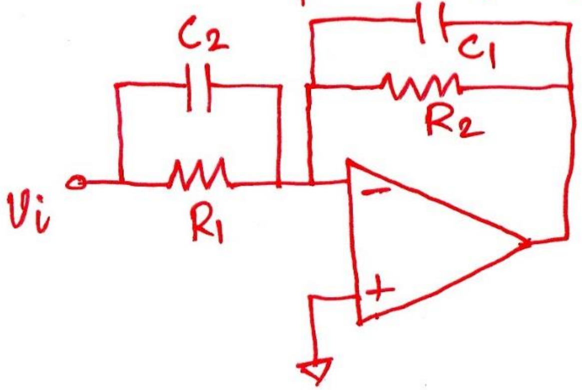
pole ω_0 = ?

zero ω_z = ?



Draw the relative positions of pole and zero in s-plane for LPF and HPF characteristic. Also, draw gain characteristic.

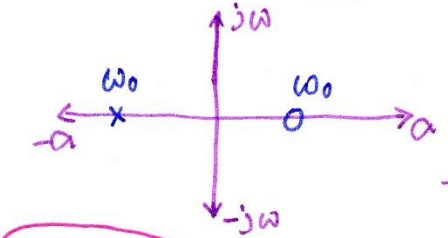
2) Active implementation :-

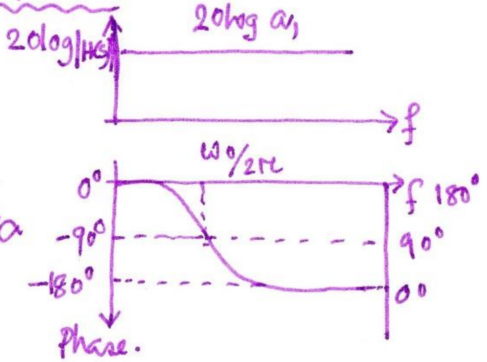


Repeat the same as asked in previous problem.

First order all-pass filter:

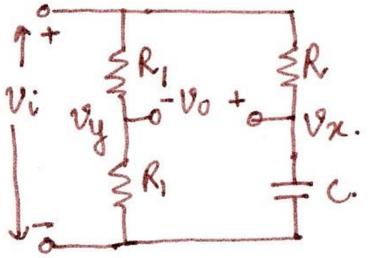
$$H(s) = \pm a_1 \frac{s-\omega_0}{s+\omega_0}$$





Phase-lag

Passive implementation:



$$v_y = \frac{v_i}{2}.$$

$$v_x = \frac{v_i}{1+sRC.}$$

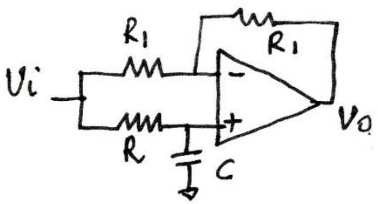
$$v_0 = v_x - v_y = \frac{v_i}{2}\left[\frac{1-sRC}{1+sRC}\right].$$

or. $H(s) = \frac{1}{2}\left[\frac{1-sRC}{1+sRC}\right].$

DC gain $= \frac{1}{2}$.

$\omega_0 = \frac{1}{RC}$, $\omega_z = -\frac{1}{RC}$, $\theta = -2\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$

Active implementation:



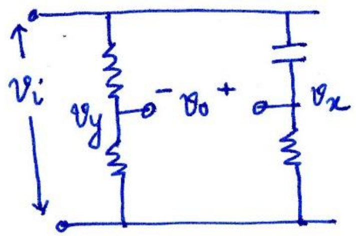
$$H(s) = \frac{1-sRC}{1+sRC}.$$

$$\theta = -2\tan^{-1}(\omega/\omega_0)$$

DC gain $=1$, $\omega_0 = -\omega_z = \frac{1}{RC}$.

Phase-lead

Passive implementation:-



$$v_y = \frac{v_i}{2}$$

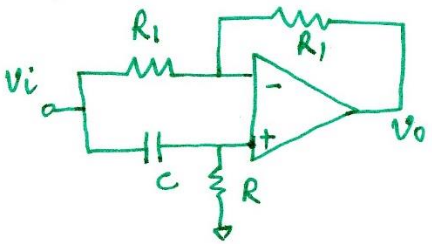
$$v_x = \frac{v_i\, sRC}{1+sRC.}$$

$$v_0 = v_x - v_y = -\frac{1}{2}\left[\frac{1-sRC}{1+sRC}\right].$$

DC gain $= \frac{1}{2}$

$\omega_0 = -\omega_z = \frac{1}{RC}$, $\theta = 180^\circ - 2\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$.

Active implementation:-



$$H(s) = -\frac{(1-sRC)}{(1+sRC)}$$

$$\theta = 180^\circ - 2\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

DC gain $=1$, $\omega_0 = -\omega_z = \frac{1}{RC}$.

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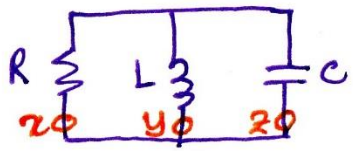
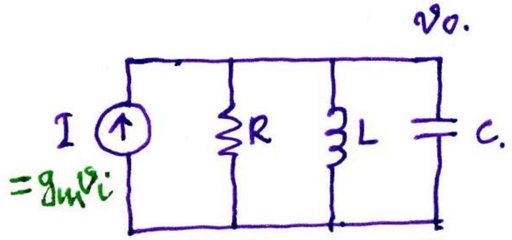
Second order filter :-

Various ways : ① use of LCR resonator ② Cascading first order filter.

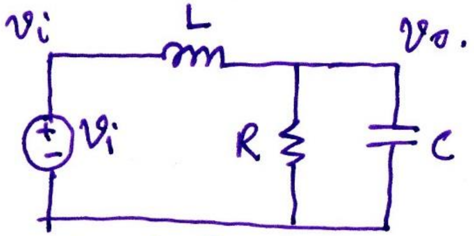
1) LCR Resonator :-



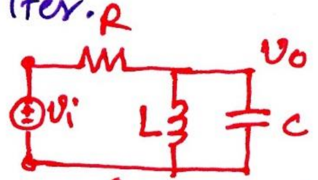
LCR resonator

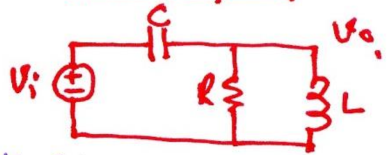


constant current excitation.



constant voltage excitation.





- Analysis with constant current excitation :-

$$I = \frac{v_0}{R} + \frac{v_0}{1/sC} + \frac{v_0}{sL}.$$

$$\text{or, } \frac{v_0}{I} = \frac{s/C}{s^2 + \frac{s}{RC} + \frac{1}{LC}} = \frac{s/C}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad \frac{\omega_0}{Q} = \frac{1}{RC} \Rightarrow Q = R\sqrt{\frac{C}{L}}.$$

- Analysis with constant voltage excitation

$$v_0 = \frac{v_i}{sL + \frac{R}{1+sCR}} \times \frac{R}{1+sCR}$$

$$\frac{v_0}{v_i} = \frac{1/LC}{s^2 + s.\frac{1}{CR} + \frac{1}{LC}} = \frac{1/LC}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

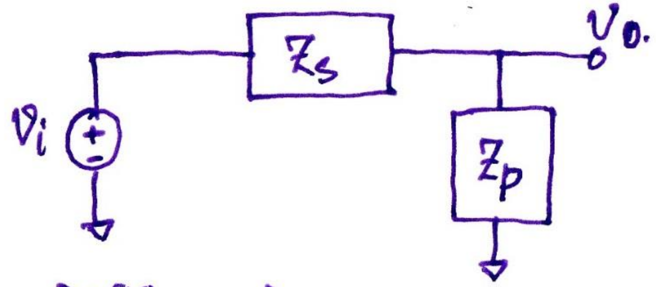
$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad \frac{\omega_0}{Q} = \frac{1}{CR} \Rightarrow Q = R\sqrt{\frac{C}{L}}.$$

- Observations :-

a) ω_0 and Q values are same in both the cases; however, the numerator is different.

b) Infact, any nodes labelled as x, y, z can be disconnected from ground and connected to v_i without altering the natural modes ω_0 and Q.. However, the numerator will be changed in these cases.

Intutive understanding of adding transmission zeros:



$$\frac{V_o}{V_i}(s) = \frac{Z_p}{Z_p + Z_s}$$

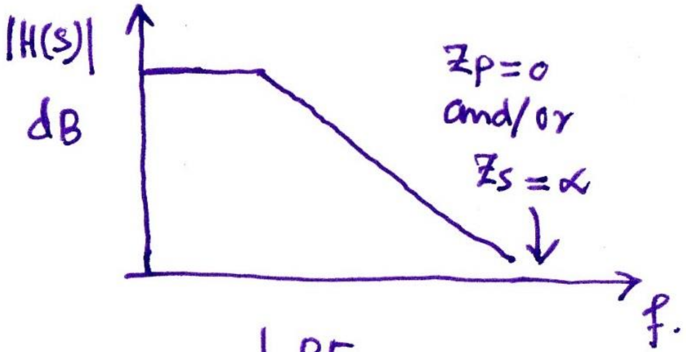
Z_p and Z_s do not go to zero/infinity simultaneously.

Different cases:-

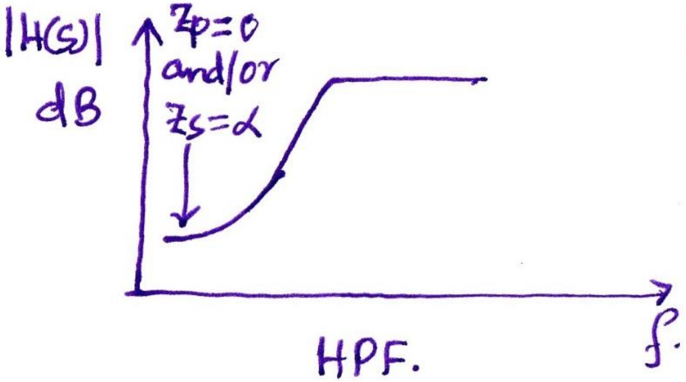
a) If at high frequency, Z_p gets shorted and/or Z_s gets infinity, it acts as LPF.

b) If at ~~high~~ low frequency, Z_s becomes infinity and/or Z_p becomes zero, it acts as HPF.

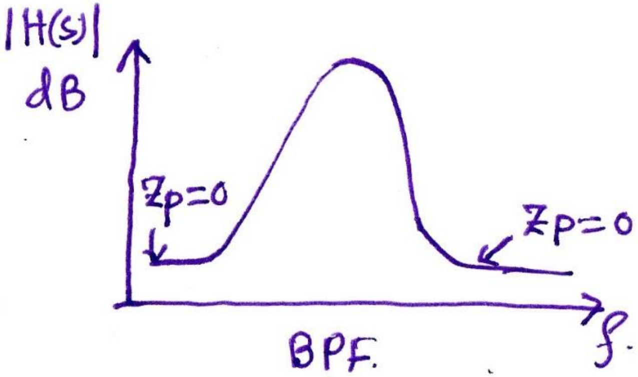
c) If Z_s remains constant, but Z_p falls to zero at both low & high frequency, then it acts as a BPF.



LPF.

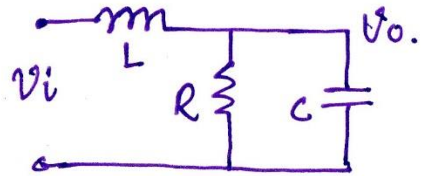


HPF.



BPF.

Realization of Lowpass filter :-



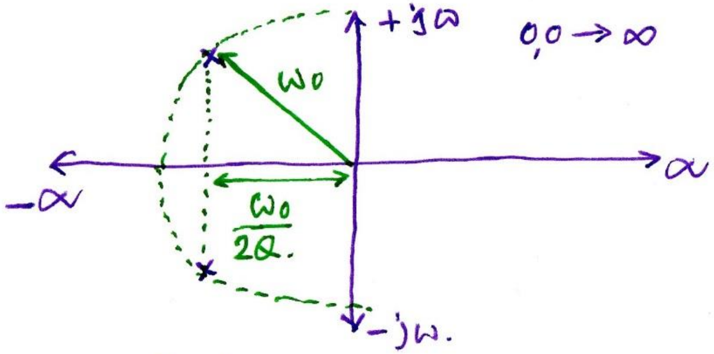
$Z_s = SL$; $s \to \infty$, $Z_L \to \infty$

$Z_p = \frac{R}{1+SCR}$; $s \to \infty$, $Z_p \to 0$.

$$H(s) = \frac{1/LC}{s^2 + \frac{s}{RC} + \frac{1}{LC}}.$$

$\omega_0 = \frac{1}{\sqrt{LC}}$, $Q = R\sqrt{C/L}$. , $|H(0)| = 1$

$$H(s) = \frac{a_0}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}.$$



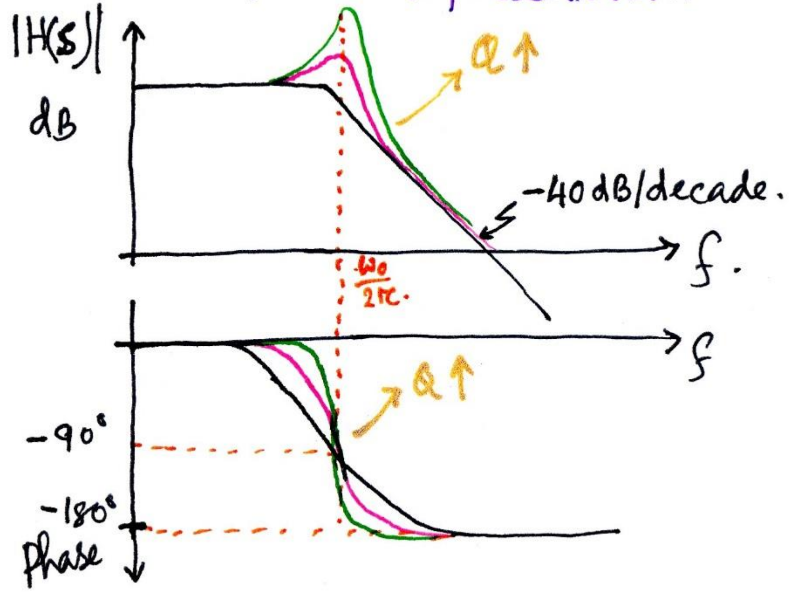
S-plane representation.

Important observation :-

a) If $Q = 0.5$, poles are real.

b) If $Q > 0.5$, poles are complex

c) The effect of Q will be exploited in filter design.



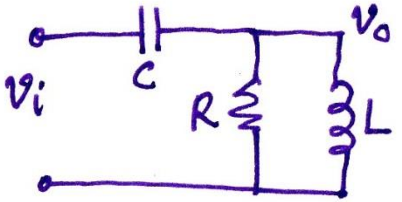
Important observation :-

a) If Q increases, the gain roll-off will be higher than -40dB/decade even in second order filter.

b) A high Q provides, better frequency selectivity, sharper transition band.

c) Play with Q values.

Realization of high pass filter :-



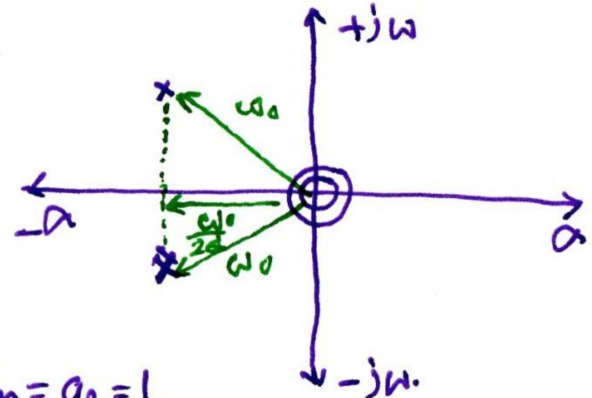
$$H(s) = \frac{s^2}{s^2 + s\left(\frac{1}{RC}\right) + \frac{1}{LC}}$$

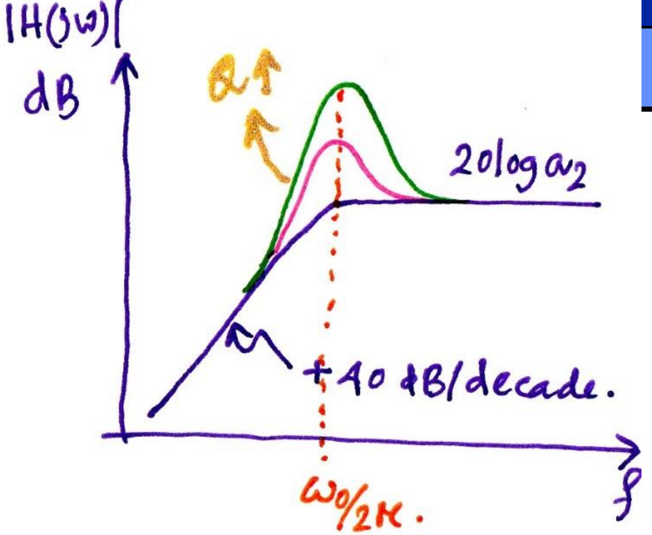
$$= \frac{a_2 s^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$s \to 0,\ Z_s \to \infty$

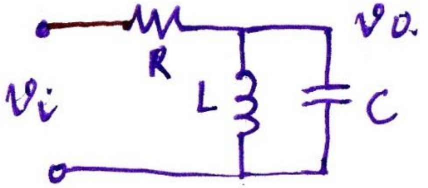
$s \to 0,\ Z_p \to 0.$

$\omega_0 = \frac{1}{\sqrt{LC}}$, $Q = R\sqrt{\frac{C}{L}}$, HF gain $= a_2 = 1.$





Realization of band-pass filter :-



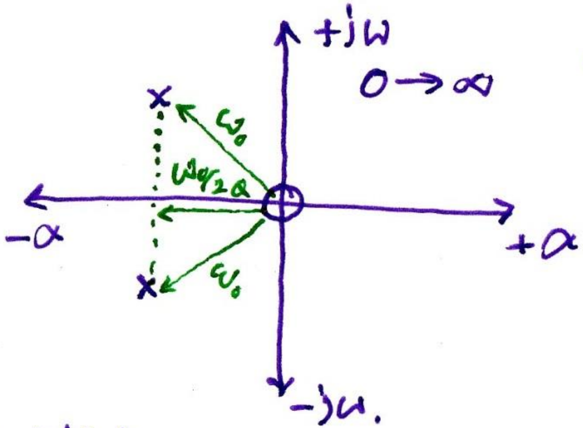
$$H(s) = \frac{s/RC}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

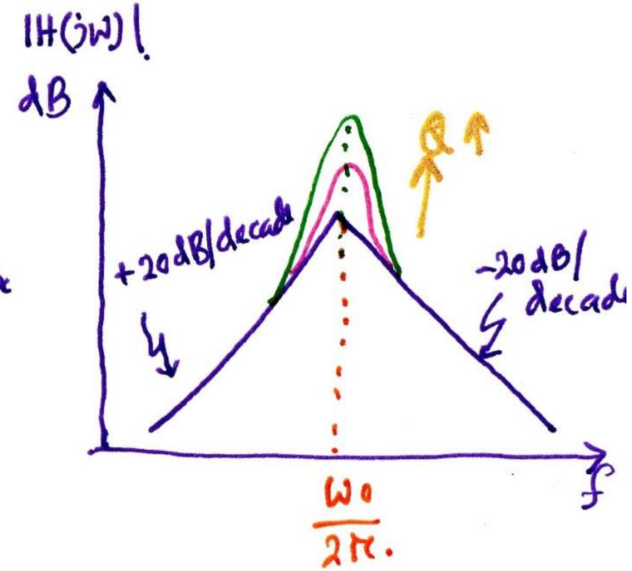
$$= \frac{a_1 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$s \to 0,\ Z_p \to 0.$

$s \to \infty,\ Z_p \to 0.$

$\omega_0 = \frac{1}{\sqrt{LC}}$, $Q = R\sqrt{\frac{C}{L}}$, Intermediate gain $= a_1 = 1.$





EE60032: Analog Signal Processing

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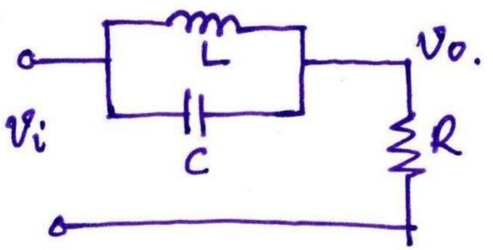
Realization of notch/band reject filter:-



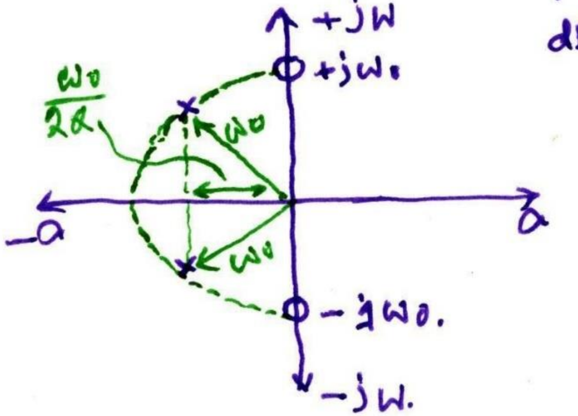
Required zeros at ω_0 to create stopband.

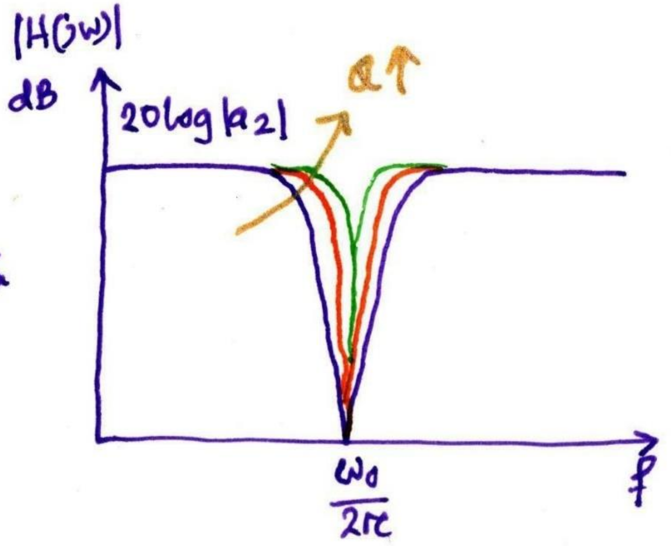
$$H(s) = \frac{s^2 + \left(\frac{1}{LC}\right)^2}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

$$= \frac{a_2\left(s^2 + \omega_0^2\right)}{s^2 + s\,\frac{\omega_0}{Q} + \omega_0^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad Q = R\sqrt{\frac{C}{L}},$$

Low and high freq gain $= a_2$





a) Here Q-factor of the poles is much lower than the same of the zeros. The Q-factor of zero $\Rightarrow \infty$

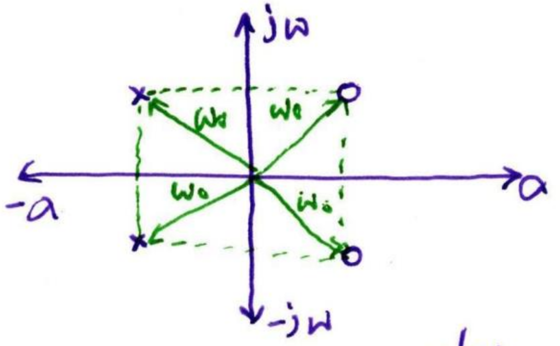
b) If the Q-factor of poles is increased the notch frequency will be more selective.

c) If the Q-factor of poles is infinity then ~~Q~~ they coincide with zeros and cancel each other without notching action.

Realization of all-pass filter :-

(a) Phase-lag filter :-

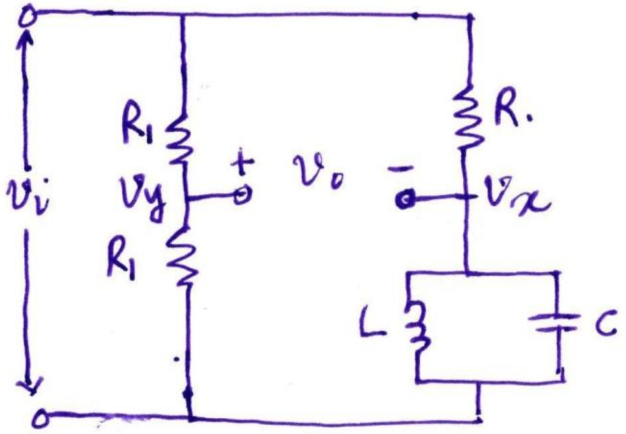
$$H(s) = \frac{a_2\left[s^2 - s\frac{\omega_0}{Q} + \omega_0^2\right]}{\left[s^2 + s\frac{\omega_0}{Q} + \omega_0^2\right]}$$ (Generalized expression)

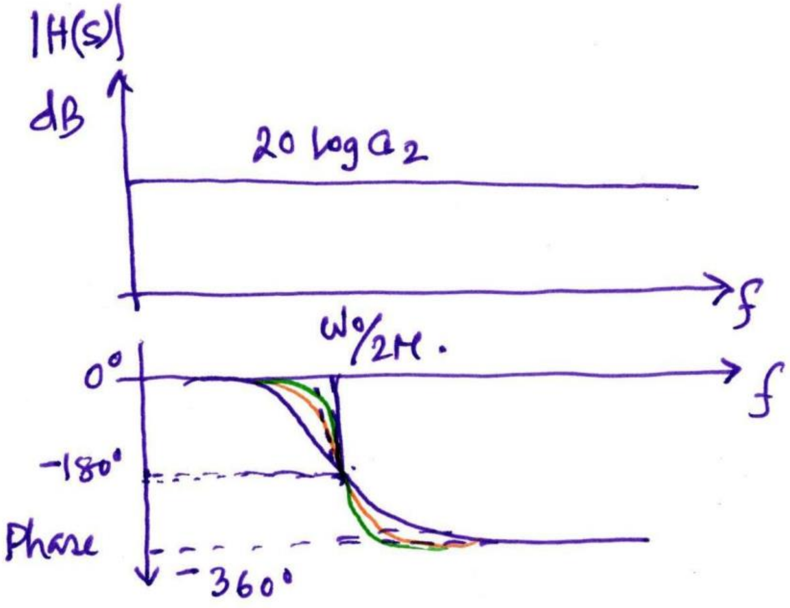


$$= 1 - \underbrace{\frac{2s\frac{\omega_0}{Q}}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}}_{BPF} \quad [\text{if } a_2 = 1]$$

$$H'(s) = \frac{H(s)}{0.5} = 0.5 - \frac{s\,\omega_0/Q}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$= (v_y - v_x)$$

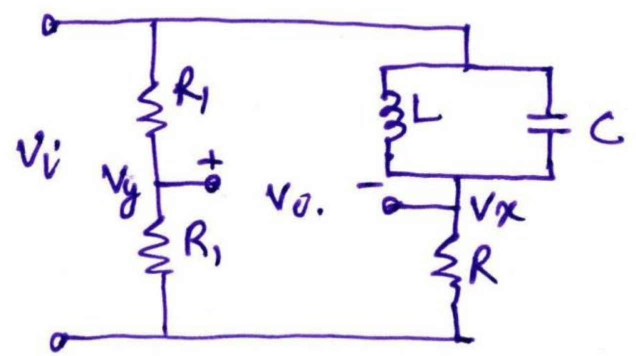




Effect of Q-factor?

* Disadvantage :-
Do not have common ground point.

b) Phase-lead filter:



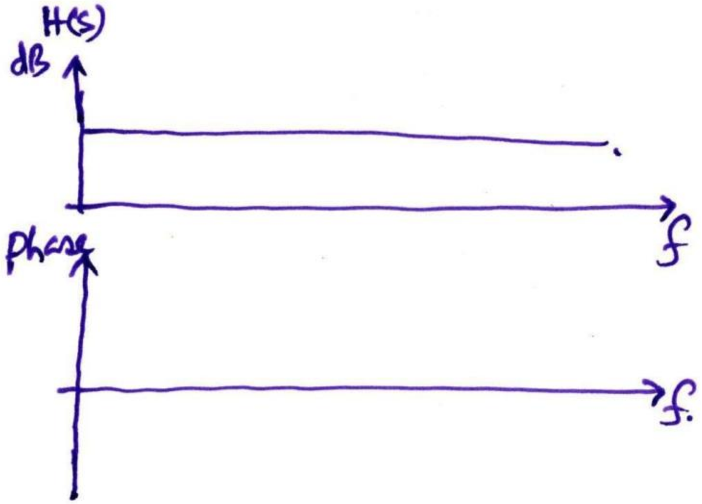
$$H(s) = 0.5 - \overbrace{\frac{s^2+\omega_0^2}{\left[s^2+s\frac{\omega_0}{Q}+\omega_0^2\right]}}^{\text{Band reject filter.}}$$

$$= -0.5\,\frac{\left[s^2 - s\,\omega_0/Q + \omega_0^2\right]}{\left[s^2+s\frac{\omega_0}{Q}+\omega_0^2\right]}$$

$$\theta = 180^\circ - 4\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

Initial phase is starting from 180°.

End phase = −180°.



Phase leading over entire range is not possible.

- Advantages of LCR resonator filter:-
  1. Suitable for high frequency application.
  2. Controlling Q-value provids additional flexibility.

- Disadvantages of LCR resonator filter:-
  1. Not suitable for low frequency application.
  2. Difficult to realize in on-chip implementation.

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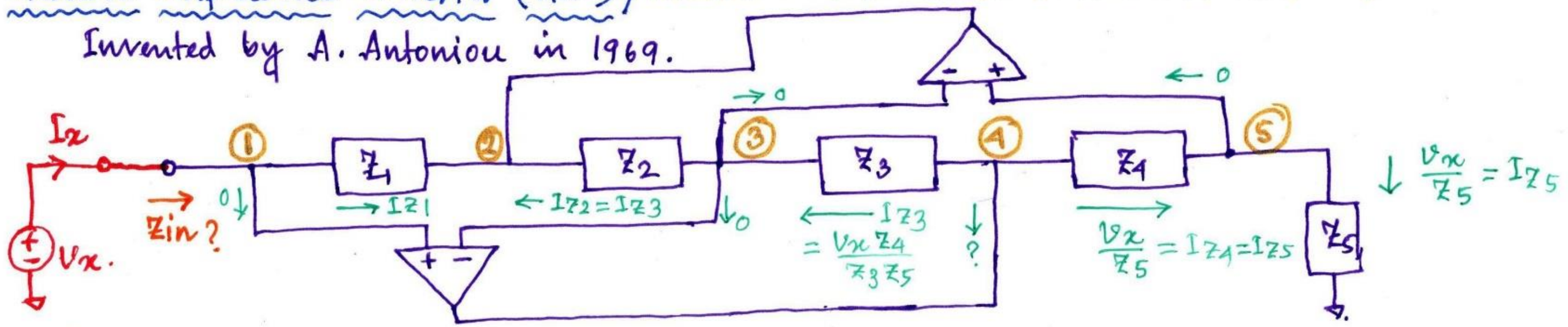
General Impedance Converter (GIC) / Antoniou Inductance Simulation Circuit:-

Invented by A. Antoniou in 1969.



$v_1 = v_3 = v_5 = v_x.$

$$v_4 = v_x + \frac{v_x}{Z_5} \cdot Z_4$$

$$I_{Z3} = \frac{v_4 - v_3}{Z_3} = \frac{v_x + v_x \cdot \frac{Z_4}{Z_5} - v_x}{Z_3}$$

$$= v_x \cdot \frac{Z_4}{Z_3 Z_5}$$

$$v_2 = v_3 - I_{Z2} \cdot Z_2 = v_3 - I_{Z3} \cdot Z_2$$

$$= v_x - \frac{v_x Z_4 Z_2}{Z_3 Z_5}$$

$$I_{Z1} = I_x = \frac{V_x - V_2}{Z_1} = \frac{V_x - V_x + \frac{V_x Z_4 Z_2}{Z_3 Z_5}}{Z_1}$$

$$= V_x \frac{Z_4 Z_2}{Z_1 Z_3 Z_5}$$

$$\boxed{Z_{in} = \frac{V_x}{I_x} = \frac{Z_1}{Z_2} \cdot \frac{Z_3}{Z_4} \cdot Z_5}$$

Observations:-

a) Based on the impedance of $Z_1 - Z_5$, Zin will change.

Examples:

i) If $Z_1 = R_1$, $Z_3 = R_3$, $Z_4 = R_4$, $Z_5 = R_5$, $Z_2 = \frac{1}{sC_2}$

then $Z_{in} = \frac{R_1}{1/sC_2} \cdot \frac{R_3}{R_4} \cdot R_5 = s\,C_2\,R_1 \cdot \frac{R_3}{R_4} \cdot R_5$

Zin becomes inductive

Always emulates grounded L

$L = C_2 R_1 \frac{R_3}{R_4} \cdot R_5$

ii) If $Z_1 = R_1$, $Z_2 = R_2$, $Z_3 = R_3$, $Z_4 = \frac{1}{sC_4}$, $Z_5 = R_5$

then $Z_{in} = \frac{R_1}{R_2} \cdot \frac{R_3}{1/sC_4} \cdot R_5 = s \frac{R_1}{R_2} \cdot R_3 C_4 \cdot R_5$

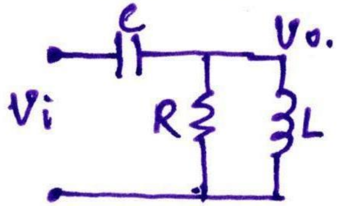
Zin becomes inductive

Emulates grounded L.

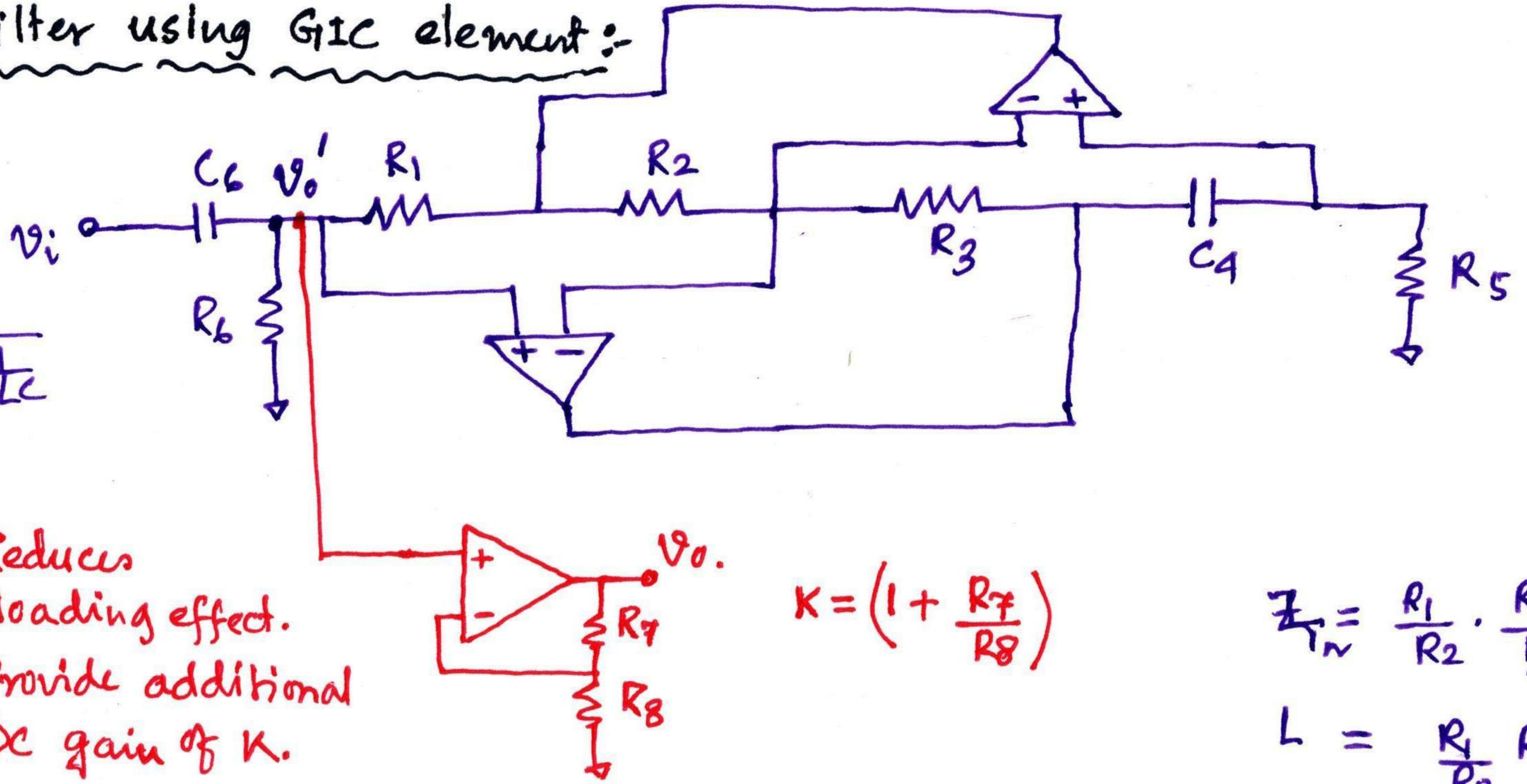
$L\ \frac{R_1}{R_2} \cdot R_3 C_4 R_5.$

iii) If $Z_1/Z_3/Z_5$ are capacitive, then Zin remains capacitive.

High pass filter using GIC element :-



$$H(s) = \frac{s^2}{s^2 + s\frac{1}{RC} + \frac{1}{LC}}$$



- Reduces loading effect.
- Provide additional DC gain of K.

$$K = \left(1 + \frac{R_7}{R_8}\right)$$

$$Z_{in} = \frac{R_1}{R_2} \cdot \frac{R_3}{1/sC_4} \cdot R_5$$

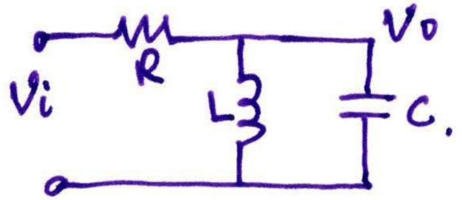
$$L = \frac{R_1}{R_2} R_3 C_4 R_5$$

$$H(s) = \frac{K s^2}{s^2 + s\frac{1}{R_6 C_6} + \frac{R_2}{C_6 R_1 R_3 C_4 R_5}}$$

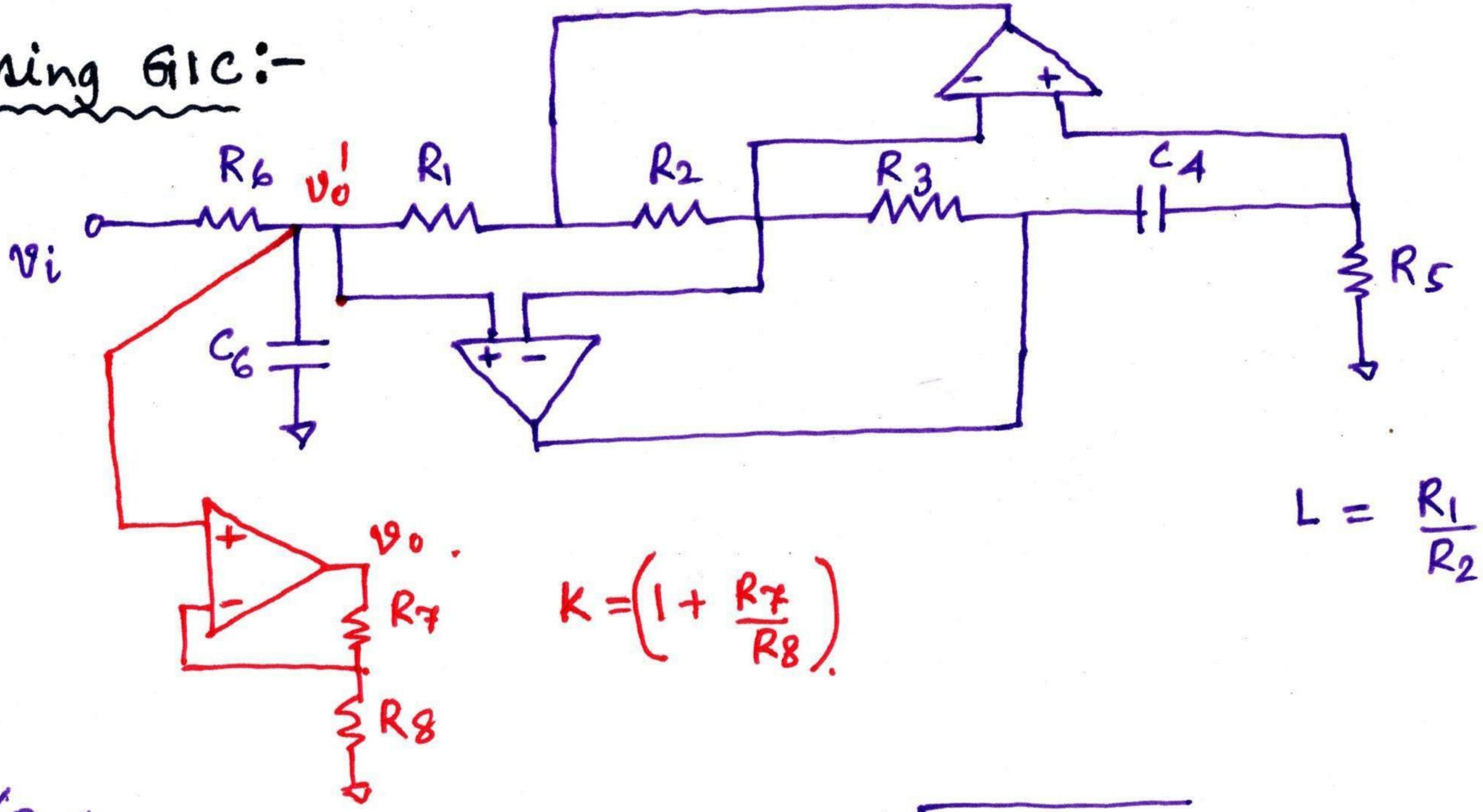
$$\omega_0 = \sqrt{\frac{R_2}{C_6 R_1 R_3 C_4 R_5}}$$

$$Q = R_6 \sqrt{\frac{C_6 R_2}{R_1 R_3 C_4 R_5}}$$

Band-pass filter using GIC:-



$$H(S) = \frac{S/RC}{s^2 + s/RC + \frac{1}{LC}}$$



$$L = \frac{R_1}{R_2} R_3 C_4 R_5$$

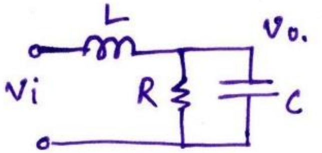
$$K = \left(1 + \frac{R_7}{R_8}\right).$$

$$H(S) = \frac{K\, S/R_6 C_6}{s^2 + \frac{s}{R_6 C_6} + \frac{R_2}{C_6 R_1 R_3 C_4 R_5}}$$

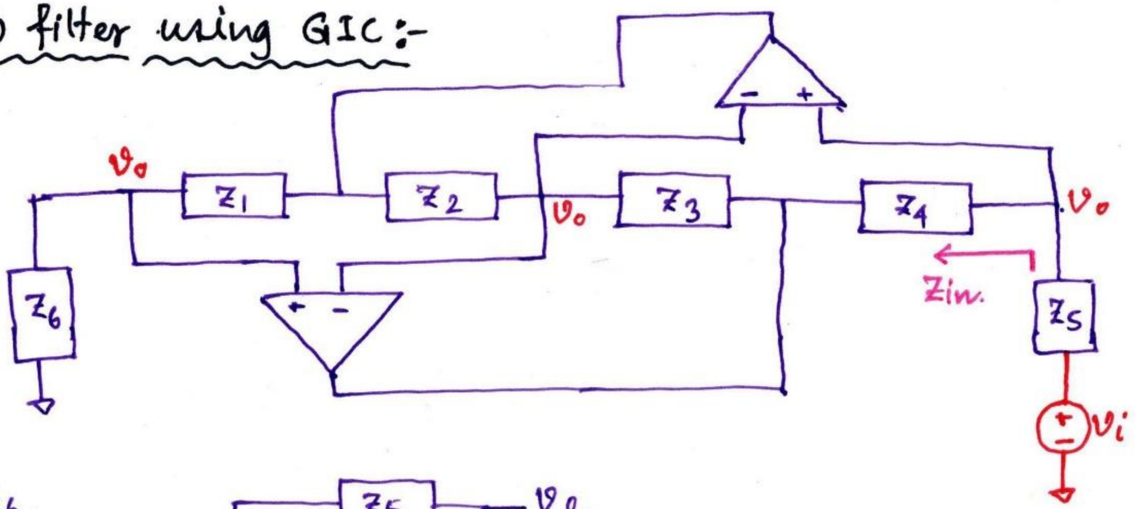
$$\omega_o = \sqrt{\frac{R_2}{C_6 R_1 R_3 C_4 R_5}}$$

$$Q = R_6 \sqrt{\frac{C_6 R_2}{R_1 R_3 C_4 R_5}}$$

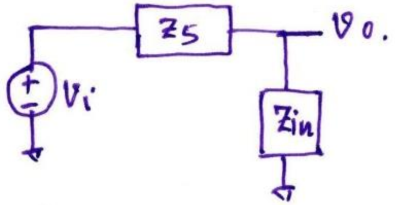
How to develop a Lowpass filter using GIC:-



$$H(s) = \frac{1/LC}{s^2 + \frac{s}{RC} + \frac{1}{LC}}.$$



$$Z_{in} = \frac{Z_4}{Z_3} \cdot \frac{Z_2}{Z_1} \cdot Z_6.$$



$$v_o = \frac{v_i Z_{in}}{Z_5 + Z_{in}}.$$

or, $$\boxed{\frac{v_o}{v_i} = \frac{Z_{in}}{Z_5 + Z_{in}}.}$$

If $Z_1 = R_1$, $Z_2 = R_2$, $Z_3 = R_3$, $Z_4 = \frac{1}{sC_4}$, $Z_5 = R_5$, $Z_6 = R_6 \| \frac{1}{sC_6}$

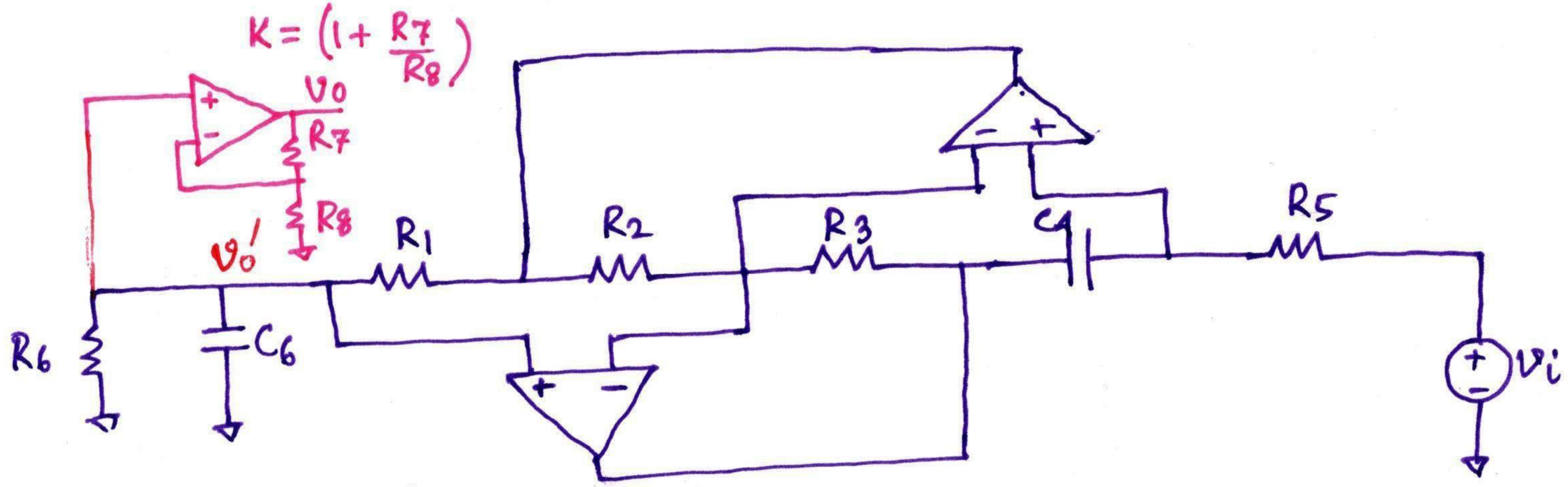
then $$Z_{in} = \frac{1}{sC_4 \cdot R_3} \times \frac{R_2}{R_1} \times \left(R_6 \| \frac{1}{sC_6}\right) = \frac{R_2 R_6}{R_1 R_3 \underbrace{sC_4 (1 + s R_6 C_6)}_{\text{Capacitive}}}$$

$$\frac{v_o}{v_{in}} = \frac{\frac{R_2 R_6}{R_1 R_3 sC_4 (1 + sC_6 R_6)}}{R_5 + \frac{R_2 R_6}{R_1 R_3 sC_4 (1 + sC_6 R_6)}} = \frac{R_2 R_6}{R_1 R_3 R_5 sC_4 (1 + sC_6 R_6) + R_2 R_6}$$

$$= \frac{R_2 R_6}{R_1 R_3 R_5 C_4 C_6 R_6 s^2 + R_1 R_3 R_5 C_4 s + R_2 R_6} = \frac{R_2 / R_1 R_3 R_5 C_4 C_6}{s^2 + \frac{s}{R_6 C_6} + \frac{R_2}{R_1 R_3 R_5 C_4 C_6}}$$

denominator is same as HPF & BPF
numerator is also same.

Low pass filter (continued) :-



$$\frac{V_o}{V_i}(s) = H(s) = \frac{K \dfrac{R_2}{R_1 R_3 R_5 C_4 C_6}}{s^2 + \dfrac{s}{R_6 C_6} + \dfrac{R_2}{R_1 R_3 R_5 C_4 C_6}}$$

$$\omega_0 = \sqrt{\frac{R_2}{R_1 R_3 R_5 C_4 C_6}}$$

$$Q = R_6 \sqrt{\frac{C_6 R_2}{R_1 R_3 C_4 R_5}}$$

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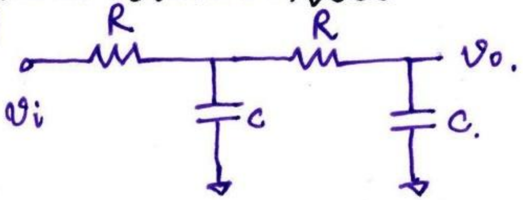
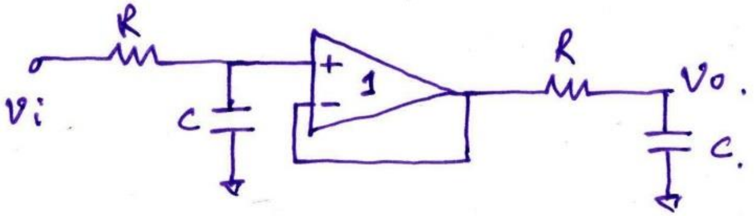
Second order filter :- Two first order filter connected in cascade.

① 

② 

Try yourself :-

$$\frac{V_o}{V_i} = \frac{1}{C^2R^2\left[s^2 + s\frac{3}{RC} + \frac{1}{R^2C^2}\right]}$$

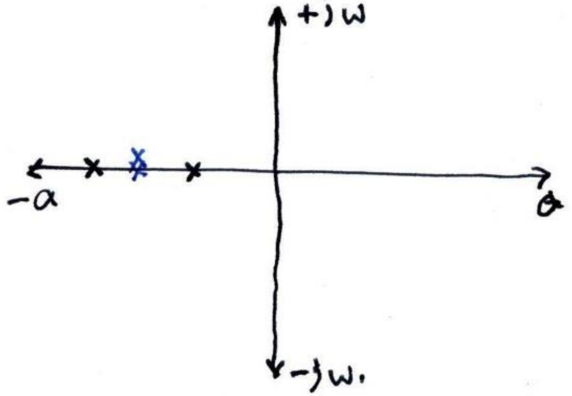
$$\frac{V_o}{V_i} = \frac{1}{C^2R^2\left[s^2 + s\frac{2}{RC} + \frac{1}{R^2C^2}\right]} = \frac{1}{(1+sRC)^2}$$

- Cascading passive filter may introduce loading effect.

$\omega_0 = \frac{1}{RC}$.

$\frac{\omega_0}{Q} = \frac{3}{RC}$.

or, $Q = \frac{1}{3} = 0.33$.



$\omega_0 = \frac{1}{RC}$

$\frac{\omega_0}{Q} = \frac{2}{RC}$

or, $Q = \frac{1}{2} = 0.5$.

Key observations :-

a) Q- factor is fixed.

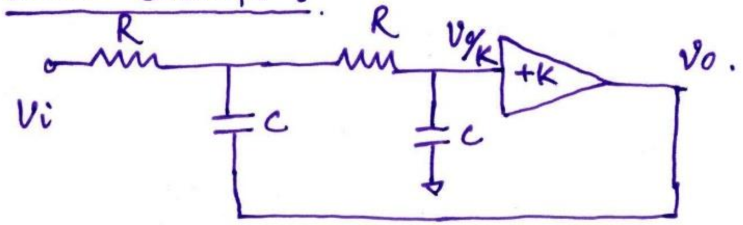
b) Q- factor also changes with loading effect.

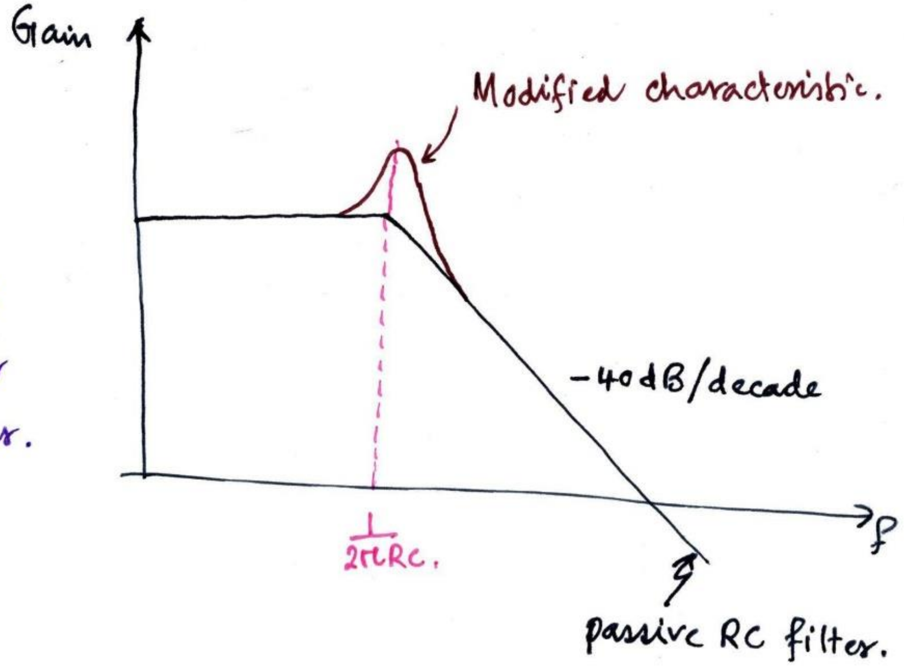
c) Poles are real ; Case 1 : two different real roots ; Case 2 : same real roots.

d) As poles are real, you can't play with Q-factor.

KRC Filter / Sallen Key filter :- Invented by Sallen-Key to improve Q-factor.

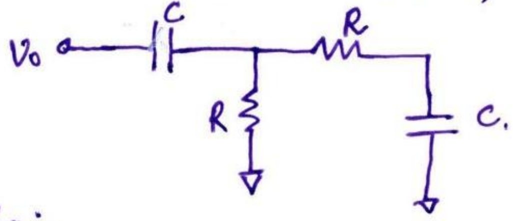
Basic concept :-

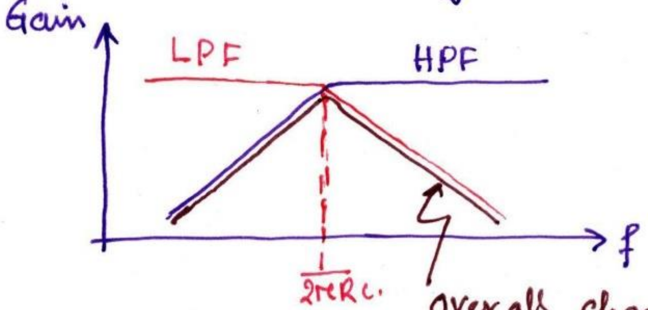




- V_i faces two first order low pass filters
- V_o faces one first order high pass filter and one first order low pass filter.

How? Assume $V_i = 0$,

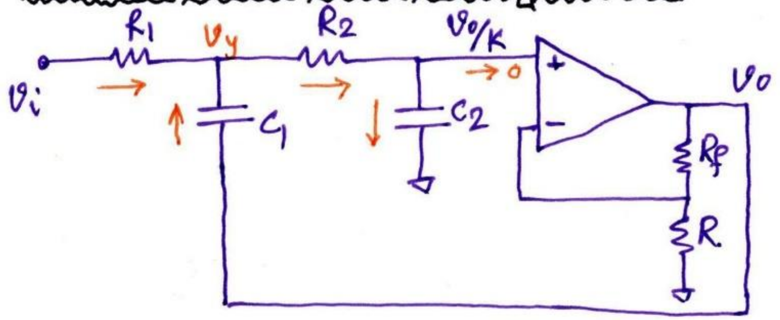




overall characteristic as first stage is HPF and second stage is LPF.

- There is a BPF characteristics from the V_o and it provides a "positive feedback" at that frequency $\frac{1}{2\pi RC}$.
- Increasing K, will alters the Q-factor.

Low pass KRC / Sallen-Key filter :-



$$K = \left(1 + \frac{R_f}{R}\right).$$

Current through $C_2 = \dfrac{v_o s C_2}{K}$

$$v_y = \frac{v_o}{K} + \frac{sC_2 v_o}{K} R_2 = \frac{v_o}{K}\left[1 + sC_2R_2\right].$$

Applying KCL at node v_y :-

$$\frac{v_i - v_y}{R_1} + (v_o - v_y) sC_1 = \frac{v_o}{K} sC_2$$

or, $v_i - \dfrac{v_o}{K}\left[1+sC_2R_2\right] + \left[v_o - \dfrac{v_o}{K}(1+sC_2R_2)\right] sC_1R_1 = \dfrac{v_o}{K} sC_2R_1$

or, $$\frac{v_o}{v_i} = \frac{K}{s^2 C_1R_1C_2R_2 + s\left[C_2R_1 + C_2R_2 + C_1R_1 - KC_1R_1\right] + 1}$$

$$= \frac{K/C_1R_1C_2R_2}{s^2 + s\dfrac{\{C_2R_1 + C_2R_2 + C_1R_1(1-K)\}}{C_1R_1C_2R_2} + \dfrac{1}{C_1R_1C_2R_2}}.$$

$$\omega_0 = \frac{1}{\sqrt{C_1R_1C_2R_2}}$$

$$Q = \frac{\sqrt{C_1R_1C_2R_2}}{C_2R_1 + C_2R_2 + C_1R_1(1-K)}$$

$$= \frac{1}{\sqrt{\dfrac{C_2R_1}{C_1R_2}} + \sqrt{\dfrac{C_2R_2}{C_1R_1}} + \sqrt{\dfrac{C_1R_1}{C_2R_2}}(1-K)}$$

Lets assume $R_1 = R_2 = R$, $C_1 = C_2 = C$ Equal components.

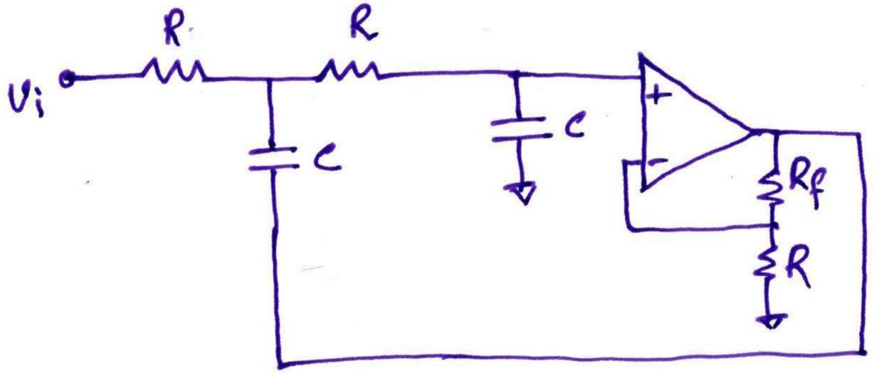
$$\omega_0 = \frac{1}{RC}.$$

$$Q = \frac{1}{2 + 1 - K} = \frac{1}{3-K}.$$

Low Pass Filter (KRC/Sallen-Key) continued :-

$$\omega_0 = \frac{1}{RC}$$

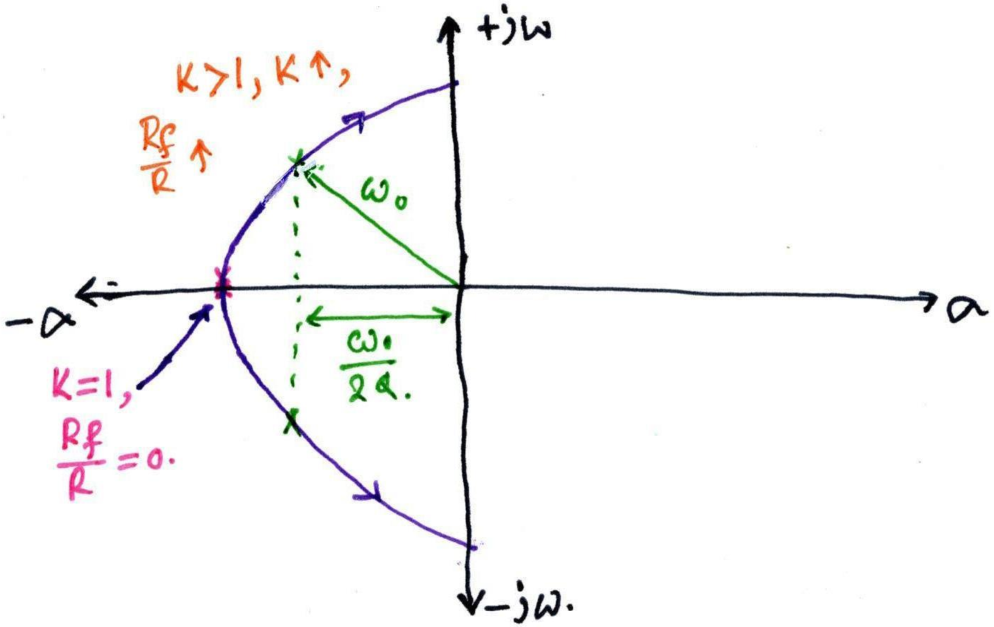
$$Q = \frac{1}{3-K}. \qquad \text{where} \quad K = \left(1 + \frac{R_f}{R}\right)$$



Various cases :-

a) If $K = 1$, $\frac{R_f}{R} = 0$, two passive low pass filters are connected in cascade condn. and op-amp is connected in unity mode, $Q = 0.5$, poles are real and at same location.

b) For $K > 1$, $Q > 0.5$, poles become complex pair. However, ω_0 remains unchanged. Filters should offer a more sharp transition.



EE60032: Analog Signal Processing

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Sensitivity analysis of SK lowpass filter:-

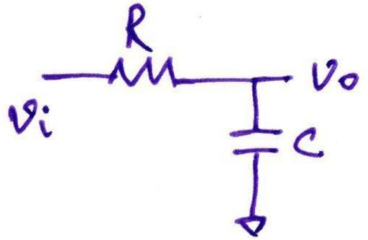
When order of the filter increases, no. of components increases.

Frequency response of analog filter depends on component values.

a) In integrated ckt, component values varry with process, voltage, temp (PVT)

b) In discrete implementation, component values change with tolerance.

Example :-



$$\omega_0 = \frac{1}{RC}$$

$$\text{or,}\quad \frac{d\omega_0}{dR} = -\frac{1}{R^2 C}$$

$$\text{or,}\quad \frac{d\omega_0}{\omega_0} = -\frac{dR}{R}$$

If there is a change ~~of~~ in +5% in R, ω_0 varries with −5%.

Definition of Sensitivity:-

Sensitivity of parameter Y with respect to the component value x is defined as

$$S_x^Y = \frac{dY/Y}{dx/x}$$

Sensitivity substantially higher than unity is undesirable.

Sensitivity analysis of SK low-pass filter (Continued):-

$$\omega_0 = \frac{1}{\sqrt{C_1 R_1 C_2 R_2}}$$

$$\text{or, } \frac{d\omega_0}{dR_1} = -\frac{1}{2}\frac{1}{R_1\sqrt{C_1 R_1 C_2 R_2}}$$

$$= -\frac{1}{2}\frac{\omega_0}{R_1}$$

$$\text{or, } \frac{d\omega_0/dR_1}{dR_1/R_1} = -\frac{1}{2}$$

$$\text{or, } S_{R1}^{\omega_0} = -0.5 < 1.$$

1% error in R_1 translates in -0.5% error in ω_0.

$$S_{R1}^{\omega_0} = S_{R2}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2}$$

$$\frac{1}{Q} = \sqrt{\frac{C_2 R_1}{C_1 R_2}} + \sqrt{\frac{C_2 R_2}{C_1 R_1}} + \sqrt{\frac{C_1 R_1}{C_2 R_2}}(1-K)$$

$$\text{or, } -\frac{dQ}{Q^2} = \frac{dR_1}{2\sqrt{R_1}}\cdot\sqrt{\frac{C_2}{C_1 R_2}} - \frac{dR_1}{2R_1}\sqrt{\frac{C_2 R_2}{C_1 R_1}} + (1-K)\frac{dR_1}{2\sqrt{R_1}}\sqrt{\frac{C_1}{C_2 R_2}}$$

$$= \frac{dR_1}{2R_1}\left[\sqrt{\frac{R_1 C_2}{C_1 R_2}} - \sqrt{\frac{C_2 R_2}{C_1 R_1}} + (1-K)\sqrt{\frac{R_1 C_1}{R_2 C_2}}\right].$$

$$\text{or, } S_{R1}^{Q} = \frac{dQ/Q}{dR_1/R_1} = -\frac{1}{2}Q\left[\sqrt{\frac{R_1 C_2}{C_1 R_2}} + \sqrt{\frac{C_2 R_2}{C_1 R_1}} + (1-K)\sqrt{\frac{R_1 C_1}{R_2 C_2}} - 2\sqrt{\frac{C_2 R_2}{C_1 R_1}}\right]$$

$$= -\frac{1}{2}Q\left[\frac{1}{Q} - 2\sqrt{\frac{C_2 R_2}{C_1 R_1}}\right].$$

$$= -\frac{1}{2} + Q\sqrt{\frac{C_2 R_2}{C_1 R_1}}$$

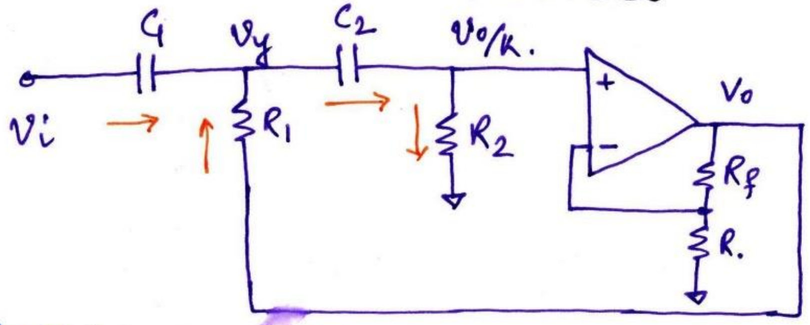
Following same procedure, we can have:-

$$S_{R2}^{Q} = -S_{R1}^{Q}$$

$$S_{C_1}^{Q} = -S_{C_2}^{Q} = -\frac{1}{2} + Q\left[\sqrt{\frac{R_1 C_2}{R_2 C_1}} + \sqrt{\frac{R_2 C_2}{R_1 C_1}}\right].$$

$$S_{K}^{Q} = QK\sqrt{\frac{R_1 C_1}{R_2 C_2}}.$$

High pass KRC/Sallen-Key filter :-



$$v_y = \frac{v_o}{K} + \frac{v_o}{KR_2} \times \frac{1}{sC_2}$$

$$= \frac{v_o}{K}\left[\frac{1+sC_2R_2}{sC_2R_2}\right].$$

Applying KCL at v_y :-

$$(v_i - v_y)sC_1 + \frac{v_o - v_y}{R_1} = \frac{v_o}{KR_2}.$$

or, $$sC_1\left[v_i - \frac{v_o}{K}\left\{\frac{1+sC_2R_2}{sC_2R_2}\right\}\right] + \frac{1}{R_1}\left[v_o - \frac{v_o}{K}\left(\frac{1+sC_2R_2}{sC_2R_2}\right)\right] = \frac{v_o}{KR_2}$$

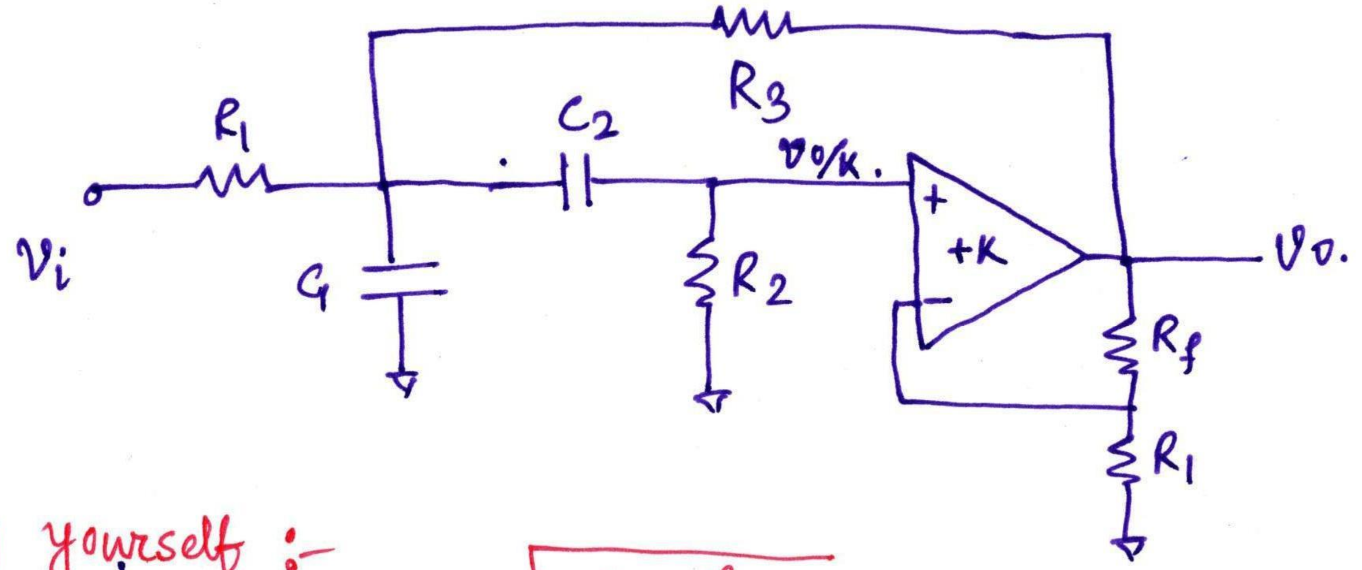
or, $$\frac{v_o}{v_i} = \frac{K s^2 C_1 C_2 R_1 R_2}{C_1 C_2 R_1 R_2\left[s^2 + \frac{s\{C_1R_1 + C_2R_1 + (1-K)C_1R_2\}}{C_1C_2R_1R_2} + \frac{1}{C_1C_2R_1R_2}\right]}$$

$$= \frac{K s^2}{s^2 + \frac{s\{C_1R_1 + C_2R_2 + (1-K)C_2R_2\}}{C_1C_2R_1R_2} + \frac{1}{C_1C_2R_1R_2}}$$

$$\omega_0 = \frac{1}{\sqrt{C_1C_2R_1R_2}} \quad ; \quad Q = \frac{1}{\sqrt{\frac{C_2R_1}{C_1R_2}} + \sqrt{\frac{C_2R_2}{C_1R_1}} + \sqrt{\frac{C_1R_1}{C_2R_2}}(1-K)}.$$

If $R_1 = R_2 = R$, $C_1 = C_2 = C$, $\omega_0 = \frac{1}{RC}$, $Q = \frac{1}{3-K}$.

KRC/Sallen Key band-pass filter :-



Try yourself :-

$$\omega_o = \sqrt{\frac{1 + R_1/R_3}{C_1 R_1 C_2 R_2}}$$

$$Q = \frac{\sqrt{1 + R_1/R_3}}{\left[1 + (1-K)\dfrac{R_1}{R_3}\right]\sqrt{\dfrac{R_2 C_2}{R_1 C_1}} + \sqrt{\dfrac{R_1 C_2}{R_2 C_1}} + \sqrt{\dfrac{R_1 C_1}{R_2 C_2}}}$$

Problem :- Determine the Q-sensitivity of the SK LPF filter for the common choice $R_1 = R_2$, and $C_1 = C_2$.

$$Q = \frac{1}{3-K}$$

$$S_{R1}^{Q} = -S_{R2}^{Q} = -\frac{1}{2} + Q = -\frac{1}{2} + \frac{1}{3-K}. \longrightarrow \left[S_{R1}^{Q} = -S_{R2}^{Q} = -\frac{1}{2} + Q\sqrt{\frac{R_2C_2}{R_1C_1}} \right].$$

$$S_{C1}^{Q} = -S_{C2}^{Q} = -\frac{1}{2} + \frac{2}{3-K}. \longrightarrow \left[S_{C1}^{Q} = -S_{C2}^{Q} = -\frac{1}{2} + Q\left(\sqrt{\frac{R_1C_2}{R_2C_1}} + \sqrt{\frac{R_2C_2}{R_1C_1}}\right) \right].$$

$$S_{K}^{Q} = QK\sqrt{\frac{R_1C_1}{R_2C_2}} = QK = \frac{K}{3-K}.$$

If $K = 1$, then $|S_{C1}^{Q}| = |S_{C2}^{Q}| = |S_{K}^{Q}| = \frac{1}{2}$ provides low sensitivity, but limited Q

Advantage of KRC/Sallen-key filter :-

a) Simple structure, only one op-amp is used.

Disadvantage of KRC/Sallen-key filter :-

a) limited Q value.

b) Sensitivity is not good,

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KHN / State variable filter :- Also known as universal filter

Invented by Kerwin, Huelsman and Newcomb in 1967.

Basic principle : Realize biquadratic transfer function by means of integrators.

Generalized transfer function of a HPF : $\frac{V_o}{V_i}(s) = \frac{\alpha s^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$.

$$\Rightarrow V_o(s)\left[1 + \frac{\omega_0}{Qs} + \frac{\omega_0^2}{s^2}\right] = \alpha V_i(s)$$

$$\Rightarrow V_o(s) = \alpha V_i(s) - \frac{\omega_0}{Qs} V_o(s) - \frac{\omega_0^2}{s^2} V_o(s)$$

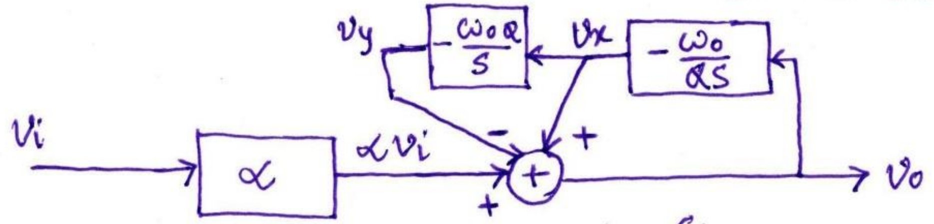
Observations :-

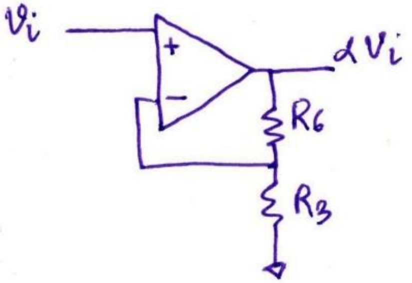
a) $V_o(s)$ can be generated by summing three terms.

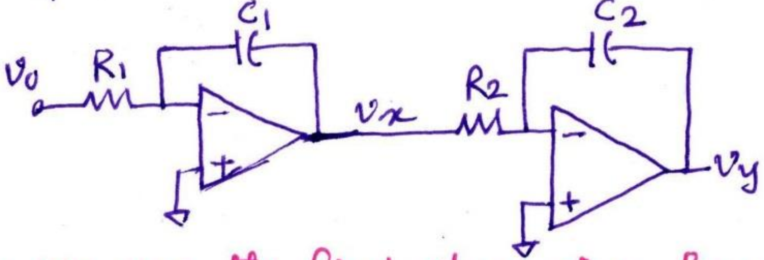
b) First term is the scaled version of v_i

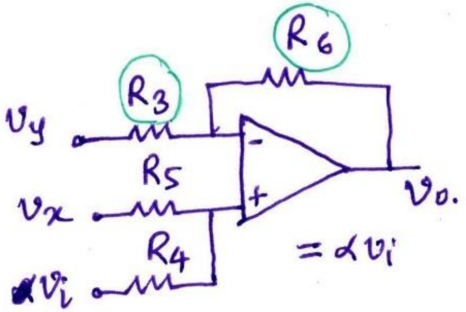
c) Second term is the integrated version of v_o

d) Third term is the double integrated version of v_o



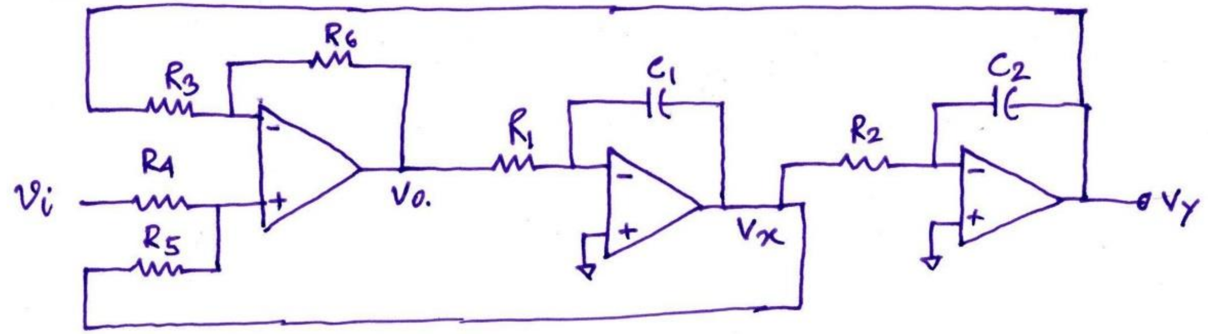






Can we use the first stage also for adding and subtracting ?

Overall implementation of KHN/state variable filter:-



$$v_x = -\frac{V_o}{R_1C_1S}$$

$$v_y = -\frac{v_x}{R_2C_2S} = \frac{V_o}{R_1R_2C_1C_2S^2}$$

Using voltage superposition, we can calculate V_o.

$$V_o = \left(1+\frac{R_6}{R_3}\right)\frac{R_5}{R_4+R_5}\cdot V_i + \left(1+\frac{R_6}{R_3}\right)\frac{R_4}{R_4+R_5}v_x - \frac{R_6}{R_3}\cdot v_y$$

$$= \left(1+\frac{R_6}{R_3}\right)\frac{R_5}{R_4+R_5}V_i - \left(1+\frac{R_6}{R_3}\right)\frac{R_4}{R_4+R_5}\cdot\frac{V_o}{R_1C_1S} - \frac{R_6}{R_3}\cdot\frac{V_o}{R_1R_2C_1C_2S^2}$$

or, $$\frac{V_o}{V_i}(s) = \frac{(1+R_6/R_3)\frac{R_5}{R_4+R_5}s^2}{s^2 + \frac{R_4}{R_5+R_4}\left(1+\frac{R_6}{R_3}\right)\frac{s}{R_1C_1} + \frac{R_6}{R_1R_2R_3C_1C_2}}$$ ⟨HPF⟩

Three cascaded stages may raise a concern of stability. Careful design and simulation are required to avoid oscillation.

$$\alpha = \left(1+\frac{R_6}{R_3}\right)\frac{R_5}{R_4+R_5}, \quad \omega_0 = \sqrt{\frac{R_6}{R_1R_2R_3C_1C_2}}, \quad Q = \frac{R_5+R_4}{R_4(R_3+R_6)}\sqrt{\frac{R_1C_1R_3R_6}{R_2C_2}}.$$

How to get BPF:-

$$\frac{V_x}{V_i}(s) = \frac{V_o}{V_i}\times\frac{V_x}{V_o} = \frac{\alpha s^2}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}\times\frac{(-1)}{R_1C_1S}$$

$$= -\frac{s\frac{\alpha}{R_1C_1}}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}$$

How to get LPF:-

$$\frac{V_y}{V_i}(s) = \frac{V_o}{V_{in}}\cdot\frac{V_y}{V_o} = \frac{\alpha s^2}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}\times\frac{1}{R_1R_2C_1C_2s^2}$$

$$= \frac{\alpha/R_1R_2C_1C_2}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}$$

Sensitivity analysis of the KHN/state variable filter:-

$$Q = \frac{R_5+R_4}{R_4(R_3+R_6)}\sqrt{\frac{R_1C_1R_3R_6}{R_2C_2}}.$$

or, $$\frac{dQ}{dR_1} = \frac{R_5+R_4}{R_4(R_3+R_6)}\cdot\sqrt{\frac{C_1R_3R_6}{R_2C_2}}\cdot\frac{1}{2\sqrt{R_1}}.$$

$$= \frac{Q}{2R_1}$$

or, $$\frac{dQ/Q}{dR_1/R_1} = \frac{1}{2} = S_{R_1}^{Q}$$

Similarly, $S_{C_1}^{Q} = \frac{1}{2}$.

$$\frac{dQ}{dR_2} = -\frac{R_5+R_4}{R_4(R_3+R_6)}\sqrt{\frac{R_1C_1R_3R_6}{C_2}}\cdot\frac{1}{2R_2\sqrt{R_2}}$$

$$= -\frac{Q}{2R_2}$$

$$S_{R_2}^{Q} = \frac{dQ/Q}{dR_2/R_2} = -\frac{1}{2}.$$

Similarly, $S_{C_2}^{Q} = -\frac{1}{2}$

$$\left|S_{R_1,R_2,C_1,C_2}^{Q}\right| = \frac{1}{2}$$

$$\frac{dQ}{dR_5} = \frac{1}{R_4(R_3+R_6)}\cdot\sqrt{\frac{R_1C_1R_3R_6}{R_2C_2}} = \frac{Q}{R_5+R_4}$$

or, $$\frac{dQ/Q}{dR_5/R_5} = \frac{R_5}{R_5+R_4} < 1.$$

or, $$S_{R_5}^{Q} = \frac{R_5}{R_5+R_4} < 1.$$

$$Q = \frac{R_5}{R_4(R_3+R_6)}\sqrt{\frac{R_1C_1R_3R_6}{R_2C_2}} + \frac{1}{(R_3+R_6)}\sqrt{\frac{R_1C_1R_3R_6}{R_2C_2}}$$

or, $$\frac{dQ}{dR_4} = -\frac{R_5}{R_4^2(R_3+R_6)}\sqrt{\frac{R_1C_1R_3R_6}{R_2C_2}} = -\frac{Q\,R_5}{R_4(R_5+R_4)}$$

or, $$\frac{\frac{dQ}{Q}}{\frac{dR_4}{R_4}} = -\frac{R_5}{R_5+R_4}$$

or, $$S_{R_4}^{Q} = -\frac{R_5}{R_5+R_4}.$$

or, $$\left|S_{R_4}^{Q}\right| < 1.$$

Sensitivity analysis of KHN/state variable filter :- (Continued)

$$Q = \frac{R_5+R_4}{R_4(R_3+R_6)}\sqrt{\frac{R_1C_1R_3R_6}{R_2C_2}}.$$

$$\frac{dQ}{dR_3} = \frac{R_5+R_4}{R_4(R_3+R_6)}\cdot\frac{1}{2\sqrt{R_3}}\sqrt{\frac{R_1C_1R_6}{R_2C_2}} - \frac{R_5+R_4}{R_4(R_3+R_6)^2}\cdot\sqrt{\frac{R_1C_1R_3R_6}{R_2C_2}}.$$

$$= \frac{Q}{2R_3} - \frac{Q}{R_3+R_6} = Q\left[\frac{R_6-R_3}{2R_3(R_3+R_6)}\right]$$

$$\frac{dQ/Q}{dR_3/R_3} = S_{R_3}^{Q} = \frac{R_6-R_3}{2(R_3+R_6)}$$

If $R_3 = R_6$, $S_{R_6}^{Q} = S_{R_3}^{Q} = 0$.

Similarly $S_{R_6}^{Q} = \dfrac{R_6-R_3}{2(R_3+R_6)}$

Advantages :-
a) KHN biquads have low sensitivity to the component value.
b) Act as an universal filter.
c) Have more independent control of filter parameters.

Disadvantages :-
a) Three op-amps in the feedback loop, which may give stability issue.
b) More components are required.

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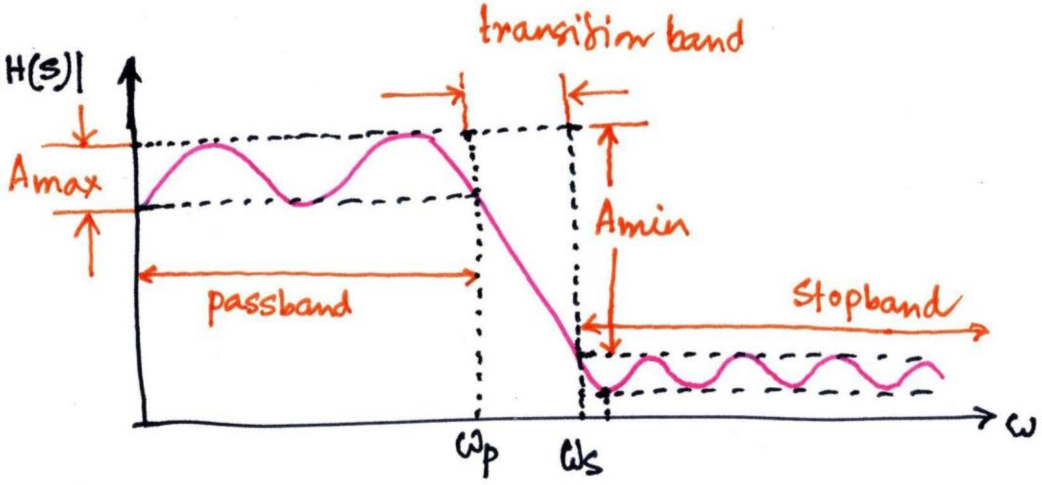
Approximation of filter functions :-

a) Based on the signal and the interferance amplitude levels, we decide stopband attenuation.

b) Depending on how close the signal frequency and interference freq. we choose the slope of transition band.



c) Depending on the nature of desired signal (audio/video), we select tolerance in the passband ripple.

Basic objective :-

a) How to determine order of the filter?

b) How to get a desired frequency response?

c) How to choose various trade-off?

This tasks are performed using approximation functions.

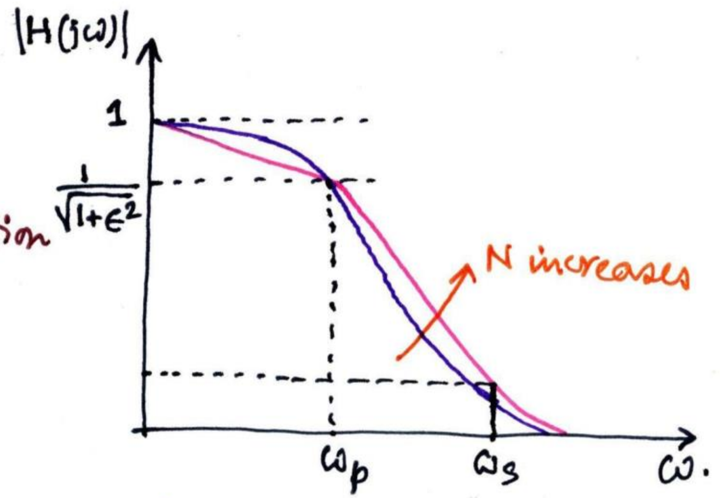
Although, these approximation functions are applied on low pass filter, they are equally applicable to develop other filter types.

Butterworth Approximation functions:- (Introduced by S. Butterworth in 1930)

- ✓ Monotonically decreasing transmission with all transmission zeros at $\omega \to \infty$.
- ✓ It provides all poles filter.
- ✓ N th order Butterworth approximation functions:-

$$|H(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2\left(\frac{\omega}{\omega_p}\right)^{2N}}}$$

where ω_p = passband freq.
ϵ = determines max. variation in pass band.



At $\omega << \omega_p$, $|H(j\omega)| = 1$.

At $\omega = \omega_p$, $|H(j\omega_p)| = \frac{1}{\sqrt{1+\epsilon^2}}$

Maximum variation in passband in dB $= 20\log\left[\frac{1}{\sqrt{1+\epsilon^2}}\right]$ dB

Passband attenuation $\rightarrow A_{max} = +10\log(1+\epsilon^2)$ dB.

Conversely, for given A_{max}, $\epsilon = \sqrt{10^{A_{max}/10} - 1}$

At $\omega = \omega_s$, $|H(j\omega_s)| = \frac{1}{\sqrt{1+\epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}}}$.

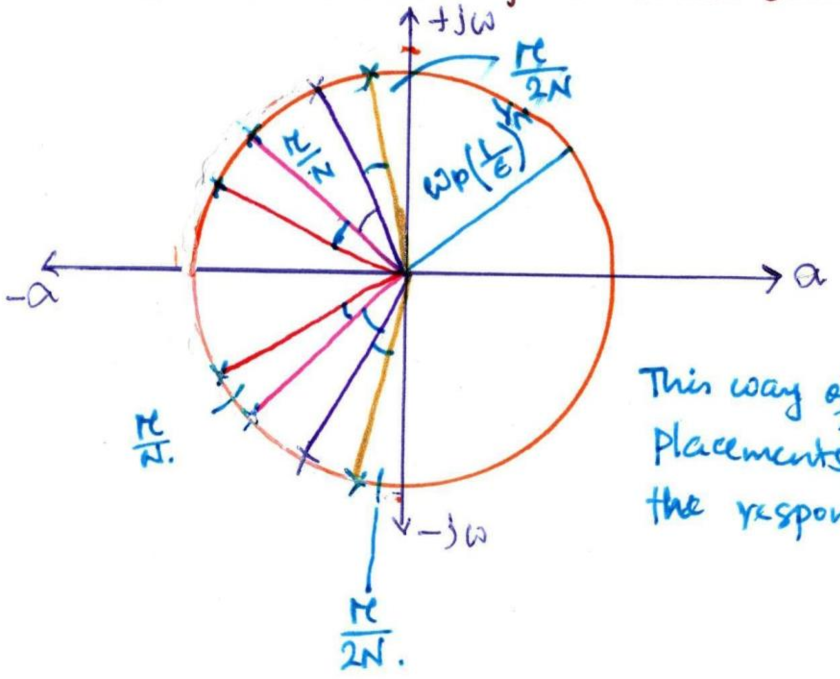
Gain $\rightarrow$ $|H(j\omega_s)|_{dB} = -10\log\left[1+\epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$.

Attenuation $\longrightarrow$ $= 10\log\left[1+\epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right] \geq A_{min}$ $\rightarrow$ provides required order N.

a) provides a flat response in passband.
b) Degree of flatness increases as N increases.
c) Provides maximally flat response.
d) As N increases, the attenuation also increases in stopband.

Butterworth Approximation functions (Continued)]:-

The natural modes of a N th order Butterworth filter can be determined graphically.



This way of pole placements optimizes the response.

$$|H(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2\left(\frac{\omega}{\omega_p}\right)^{2N}}} = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_p\left(\frac{1}{\epsilon}\right)^{1/N}}\right)^{2N}}}$$

$$\omega_0 = \omega_p\left(\frac{1}{\epsilon}\right)^{1/N}.$$

$$\omega_{p1,2} = \omega_0\left[\cos\theta \pm j\sin\theta\right]$$

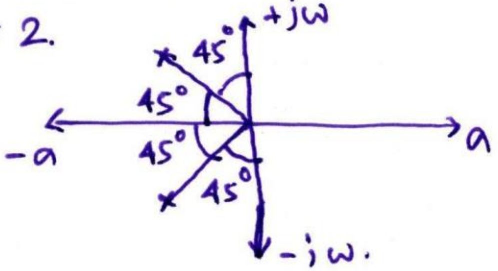
$$\omega_{p3,4} = \text{- - -}$$

$$H(j\omega) = \frac{K\,\omega_0^{N}.}{(s+\omega_{p1})(s+\omega_{p2})\ldots(s+\omega_{pn})}$$

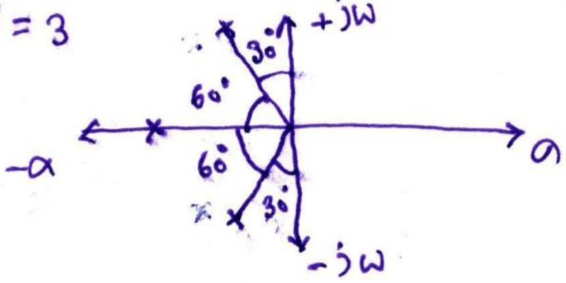
K= constant gain

Few examples :-

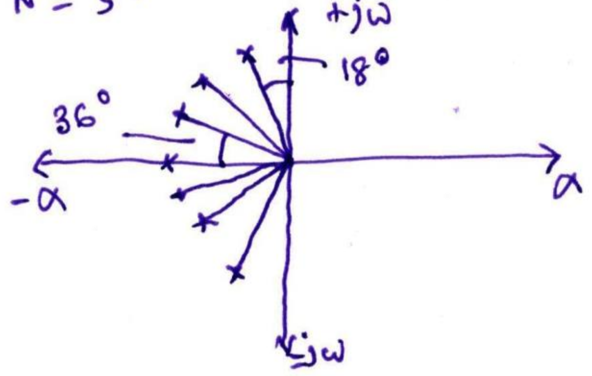
1) N= 2.



2) N = 3



3) N = 5



Example :- Find the Butterworth transfer function that meets the following low pass filter specifications: $f_p = 10$ KHz, $A_{max} = 1$ dB, $f_s = 15$ KHz, $A_{min} = 25$ dB, dc gain $K = 1$.

$$A_{max} = 10 \log(1+\epsilon^2) = 1.$$

or, $\log(1+\epsilon^2) = 0.1 \Rightarrow \epsilon = 0.5088$

$$A_{min} = 10 \log\left[1+\epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right] = 25.$$

or, $\log\left[1+\epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right] = 2.5.$

or, $1+\epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N} = 10^{2.5} = 316.22$

or, $\epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N} = 315.22.$

or, $\left(\frac{\omega_s}{\omega_p}\right)^{2N} = 1217.7$

or, $(1.5)^{2N} = 1217.7$

For $N = 8$, $(1.5)^{16} = 656.8$

For $N = 9$, $(1.5)^{18} = 1477.9.$

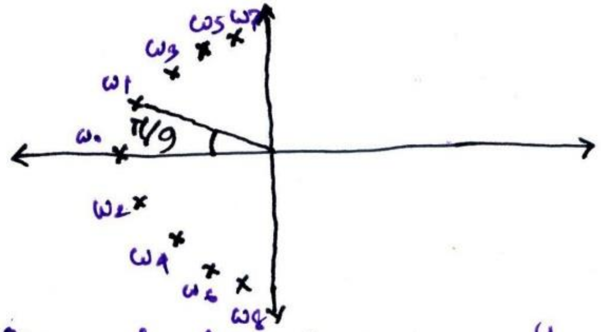
So, $\boxed{N \approx 9.}$

$$\omega_0 = \omega_p \left(\frac{1}{\epsilon}\right)^{1/N}.$$

$$= 2\pi \cdot f_p \left(\frac{1}{\epsilon}\right)^{1/N}.$$

$$= 2\pi \cdot 10K \left(\frac{1}{0.5088}\right)^{1/9}$$

$$= 6.733 \times 10^4 \text{ rad/S.}$$



One real pole $\omega_0 = 6.733 \times 10^4$ rad/S.

$\omega_{1,2} = \omega_0 (\cos 20^\circ \pm j \sin 20^\circ)$

$\omega_{3,4} = \omega_0 (\cos 40^\circ \pm j \sin 40^\circ)$

$\omega_{5,6} = \omega_0 (\cos 60^\circ \pm j \sin 60^\circ)$

$\omega_{7,8} = \omega_0 (\cos 80^\circ \pm j \sin 80^\circ)$

$$H(s) = \frac{\omega_0^9}{(s+\omega_0)(s+\omega_1)(s+\omega_2)\cdots(s+\omega_8)}$$

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(iii) The Chebyshev Approximation function :- (P. L. Chebyshev introduced in 1899)

* Exhibits an equiripple response in passband.
* Monotonically decreasing transmission in stopband.

The magnitude response of a Chebyshev fn. is.

$$|H(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2 \cos^2\left\{N \cos^{-1}\left(\frac{\omega}{\omega_p}\right)\right\}}} \quad \text{for } \omega \le \omega_p$$

and

$$|H(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2 \cosh^2\left\{N \cosh^{-1}\left(\frac{\omega}{\omega_p}\right)\right\}}} \quad \text{for } \omega \geq \omega_p$$

At passband, $\omega = \omega_p$

$$|H(j\omega_p)| = \frac{1}{\sqrt{1+\epsilon^2}}$$ Same as Butterworth fn.

$$A_{max} = 10 \log(1+\epsilon^2) \Rightarrow \epsilon = \sqrt{10^{A_{max}/10} - 1}$$

At stopband, $\omega = \omega_s$.

$$|H(j\omega_s)| = 10 \log\left[1+\epsilon^2 \cosh^2\left\{N \cosh^{-1}\left(\frac{\omega_s}{\omega_p}\right)\right\}\right]$$

Order of the filter will be decided based on the attenuation requirement in the stopband.

The poles of the Chebyshev filter :-

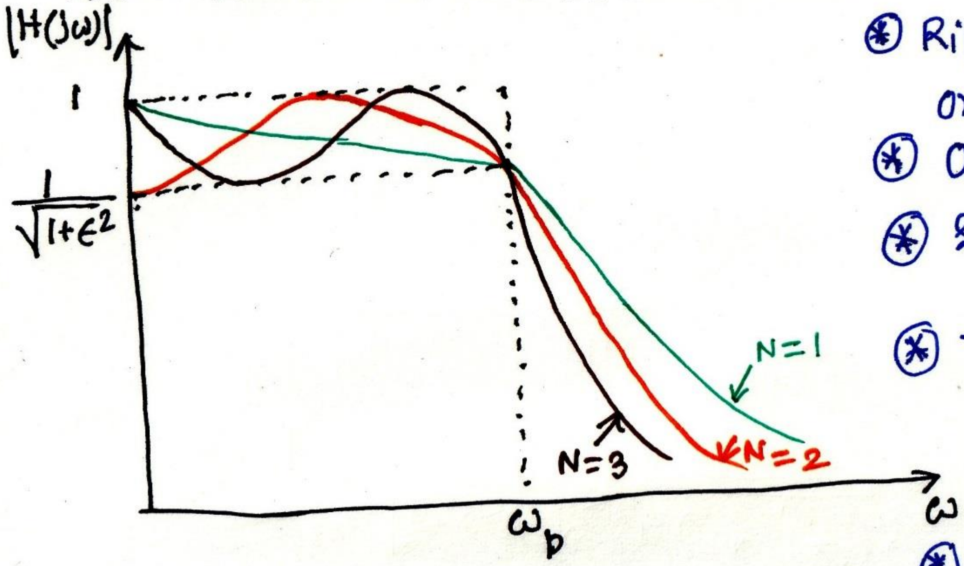
$$\omega_k = -\omega_p \sin\left[\frac{2K-1}{N} \cdot \frac{\pi}{2}\right] \sinh\left[\frac{1}{N} \cdot \sinh^{-1} \frac{1}{\epsilon}\right] + j\,\omega_p \cos\left[\frac{2K-1}{N} \cdot \frac{\pi}{2}\right] \cosh\left[\frac{1}{N} \cdot \sinh^{-1} \frac{1}{\epsilon}\right]$$

where $K = 1, 2, 3, \ldots, N$.

Overall filter transfer fn.

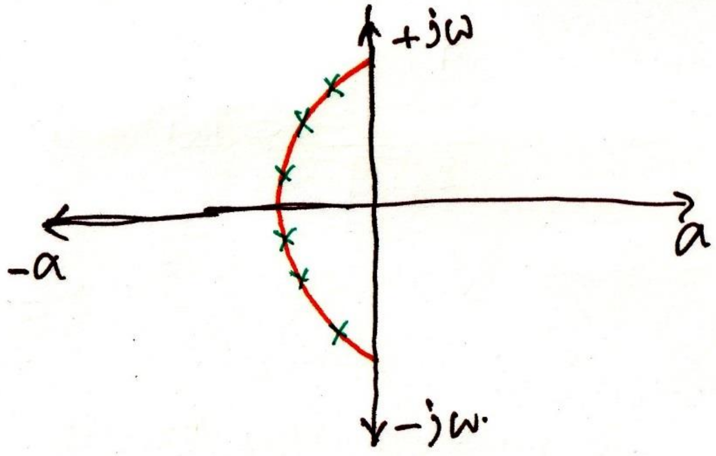
$$H(s) = \frac{\omega_p^{N}}{\epsilon\, 2^{N-1} (s+\omega_1)(s+\omega_2) \cdots (s+\omega_n)}$$

The Chebyshev Approximation function (Continued)



* Ripples confined in a band, does not change with order.
* Odd oder provides $|H(0)| = 1$.
* Even order provides $|H(0)| = \frac{1}{\sqrt{1+\epsilon^2}}$, max. deviation at ω_0.
* Total no. of passband maxima and minima equals to the order of the filter N.
* All zeros are placed at ∞, all pole filter.

*



Here, poles move in elliptical path

(In Butterworth, poles move in circular path)

* Chebyshev provides a better approximation than the Butterworth fn.
* Chebyshev provides a greater stopband attenuation than the Butterworth filter if their order are same.
* For same attenuation in stopband, Chebyshev requires lower order than the Butterworth fn.

Problems :- Find the Chebyshev approximation functions that meets the low-pass filter specifications : $f_p = 10$ KHz, $A_{max} = 1$ dB, $f_s = 15$ KHz, $A_{min} = 25$ dB, dc gain = 1.

$$\epsilon = \sqrt{10^{A_{max}/10} - 1} = \sqrt{10^{1/10} - 1} = 0.5088.$$

At stop band $\omega = \omega_s$,

$$|H(j\omega_s)| = 10 \log\left[1 + \epsilon^2 \cosh^2\left\{N \cosh^{-1}\left(\frac{\omega_s}{\omega_p}\right)\right\}\right] = 25.$$

or, $10 \log\left[1 + (0.5088)^2 \cosh^2\left\{N \cosh^{-1}(1.5)\right\}\right] = 25.$

or, $\log\left[1 + 0.2589 \cosh^2\{N \times 0.9624\}\right] = 2.5.$

or, $\cosh^2\{N \times 0.9624\} = 1217.565.$

$N = 4.411$

The required order of the Chebyshev fn. will be 5 (nearest higher integer).

For same attenuation, the order of Butterworth fn. was 9. Whereas in Chebyshev approximation, the order becomes 5 at the expense of equiripple.

$$\omega_K = -\omega_p \sin\left[\frac{2K-1}{N}\cdot\frac{\pi}{2}\right] \sinh\left[\frac{1}{N}\sinh^{-1}\left(\frac{1}{\epsilon}\right)\right] + j\omega_p \cos\left[\frac{2K-1}{N}\cdot\frac{\pi}{2}\right] \cosh\left[\frac{1}{N}\cosh^{-1}\left(\frac{1}{\epsilon}\right)\right].$$

For $K = 1$, $\frac{2K-1}{N}\cdot\frac{\pi}{2} = \frac{1}{5}\cdot\pi/2$, For $K = 5$, $\frac{2K-1}{N} \times \frac{\pi}{2} = \frac{9}{5}\times\frac{\pi}{2} = \pi - \frac{1}{5}\pi/2$

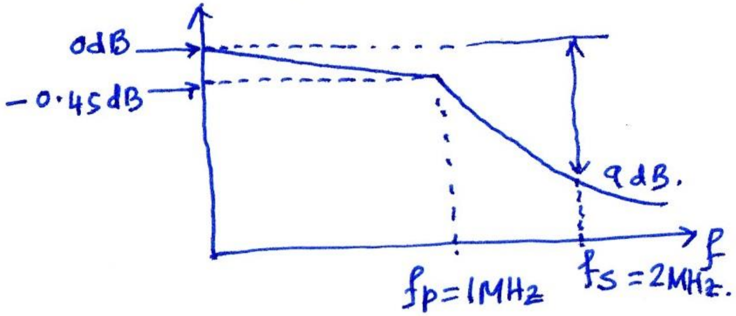
$\omega_{p1} = \omega_p[-0.0893 + j\,0.9833]$ and $\omega_{p5} = \omega_p[-0.0893 - j\,0.9833]$.

$\omega_{p2} = \omega_p[-0.2342 + j\,0.6199]$ and $\omega_{p4} = \omega_p[-0.2342 - j\,0.6199]$.

$\omega_{p3} = -\omega_p \times 0.289$ as $\frac{2K-1}{N}\cdot\pi/2 = \pi/2$

Problem :- A low pass filter must provide a passband flatness of 0.45 dB for $f_p = 1\,MHz$ and a stopband attenuation of 9 dB at $f_s = 2\,MHz$. Determine the order of the Butterworth approximation ~~satis~~ satisfying these requirements. Using a Sallen-Key topology as core, design the Butterworth approximation fns.

$$\epsilon = \sqrt{10^{A_{max}/10} - 1} = \sqrt{10^{0.045} - 1} = 0.3303.$$



$$10 \log \left[1 + \epsilon^2 \left(\frac{\omega_s}{\omega_p}\right)^{2N}\right] = A_{min}$$

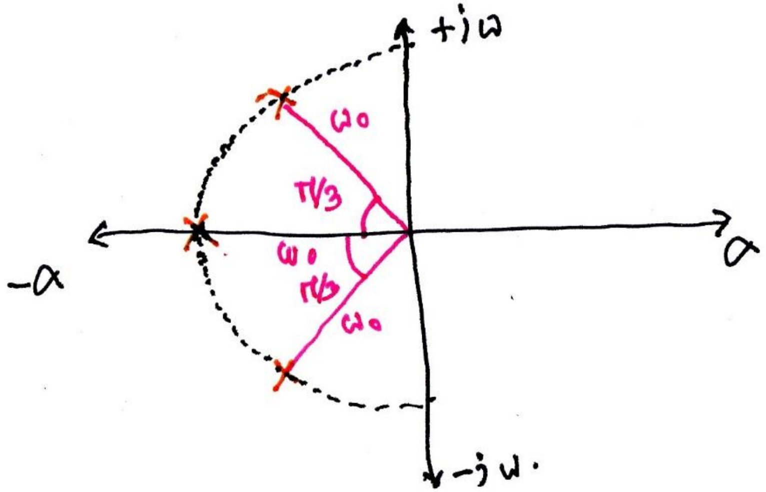
or, $$\log\left[1 + (0.3303)^2 (2)^{2N}\right] = 0.9.$$

or, $$(2)^{2N} = 63.64$$

or, $$4^N = 63.64 \qquad 4^3 = 64.$$

or, $$N \approx 3.$$

Minimum 3rd order filter is required.
1 real pole and one complex pole pair.



$$\omega_0 = \frac{\omega_p}{\epsilon^{1/N}} = \frac{2\pi \cdot f_p}{\epsilon^{1/N}} = \frac{2\pi \times 1M}{(0.3303)^{1/N}}.$$

$$= 2\pi\,(1.45\,MHz)$$

$$f_0 = 1.45\,MHz.$$

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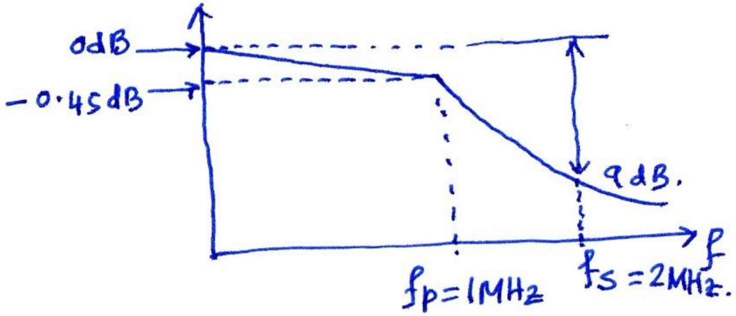

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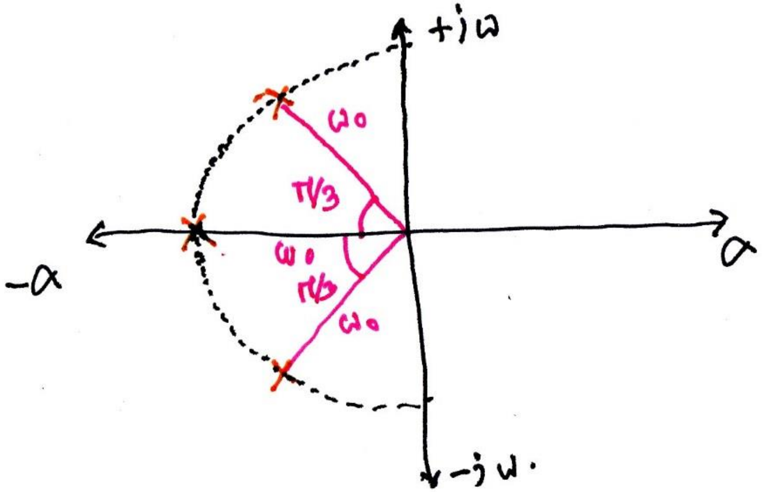
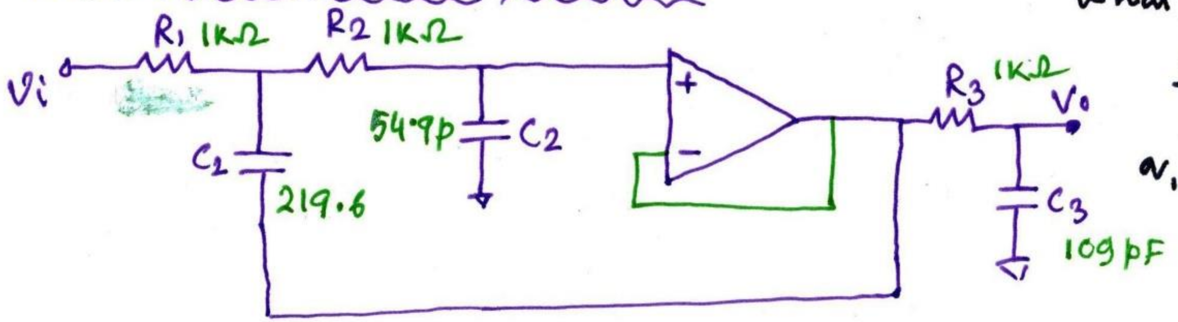


$$\omega_0 = \frac{\omega_p}{\epsilon^{1/N}} = \frac{2\pi \cdot f_p}{\epsilon^{1/N}} = \frac{2\pi \times 1M}{(0.3303)^{1/N}}.$$

$= 2\pi (1.45 \text{ MHz})$

$f_0 = 1.45$ MHz.

Sallen-Key filter implementation :-



What is the Q-factor required?

$$\frac{\omega_0}{2Q} = \omega_0 \cos 60^\circ$$

or, $\frac{1}{2Q} = \frac{1}{2} \Rightarrow Q = 1.$

Different design strategies can be adopted :-

1) For equal component choice, $R_1 = R_2$, $C_1 = C_2$, $Q = \frac{1}{3-K}$. If $K = 2$, $Q = 1$. Try yourself.

2) For equal resistance choice, $R_1 = R_2$, $C_1 \neq C_2$, $K = 1$.

3) For equal capacitance choice, $C_1 = C_2$, $R_1 \neq R_2$, $K = 1$.

→ Generalised expression of $Q = \dfrac{1}{\sqrt{\frac{C_2 R_1}{C_1 R_2}} + \sqrt{\frac{C_2 R_2}{C_1 R_1}} + \sqrt{\frac{C_1 R_1}{C_2 R_2}}\,(1-K)}$

If $K = 1$, $Q = \dfrac{1}{\sqrt{\frac{C_2 R_1}{C_1 R_2}} + \sqrt{\frac{C_2 R_2}{C_1 R_1}}}$

If $R_1 = R_2 = R$, $Q = \dfrac{1}{2\sqrt{C_2/C_1}} = 1$

or, $\frac{1}{2}\sqrt{\frac{C_1}{C_2}} = 1$

or, $\boxed{C_1 = 4 C_2}$

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{R\sqrt{4 C_2^2}} = \frac{1}{2RC_2} = 2\pi \times 1.45\,\text{MHz}$$

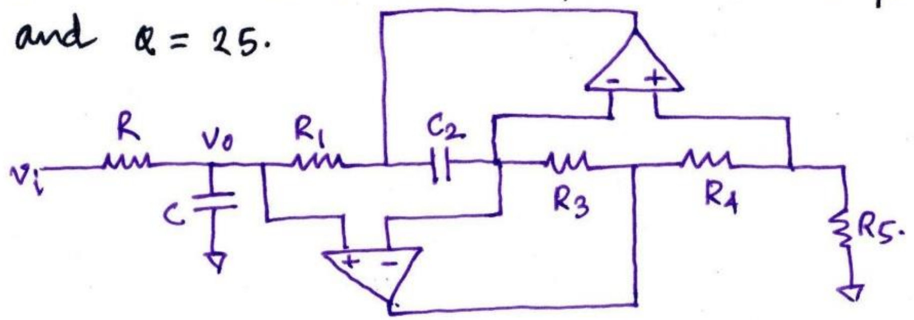
If $R = 1\,\text{k}\Omega$, $C_2 = \frac{1}{2\pi \times 1.45\text{M} \times 1\text{K} \times 2} = 54.9\,\text{pF}$

$C_1 = 4 C_2 = 4 \times 54.9\,\text{pF} = 219.6\,\text{pF}.$

$$\omega_3 = \frac{1}{R_3 C_3} = 2\pi \times 1.45\,\text{M}$$

If $R_3 = 1\,\text{k}\Omega$, $C_3 = \frac{1}{2\pi \times 1.45\text{M} \times 1\text{K}.} = 109\,\text{pF}.$

Problem: Draw a second order bandpass filter using GIC block. Derive its transfer function. Find out the components values for a band-pass response with $f_0 = 100$ kHz, and $Q = 25$.



$$SL = \frac{R_1}{1/sC_2} \cdot \frac{R_3}{R_4} \cdot R_5$$

$$= \frac{s C_2 R_1 R_3 R_5}{R_4}$$

$$L = \frac{C_2 R_1 R_3 R_5}{R_4}.$$

If $R_1 = R_3 = R_5 = R_4 = R_x$.

$$L = R_x^2 C_2.$$

$$\frac{V_o}{V_i} = H(s) = \frac{s/RC}{s^2 + s/RC + \frac{1}{LC}}.$$

$$\frac{\omega_0}{Q} = \frac{1}{RC}$$

or, $\frac{2\pi \times 100K}{25} = \frac{1}{RC}.$

Assuming $C = 1\,nF$, $R = 39.79\,K\Omega$.

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \frac{1}{\sqrt{LC}} = 2\pi(100K)$$

or, $\frac{1}{\sqrt{L.1n}} = 2\pi(100K)$

$\Rightarrow L = \frac{1}{1n\{2\pi(100K)\}^2}$

or, $L = 2.533\,mH.$

$$L = R_x^2 C_2.$$

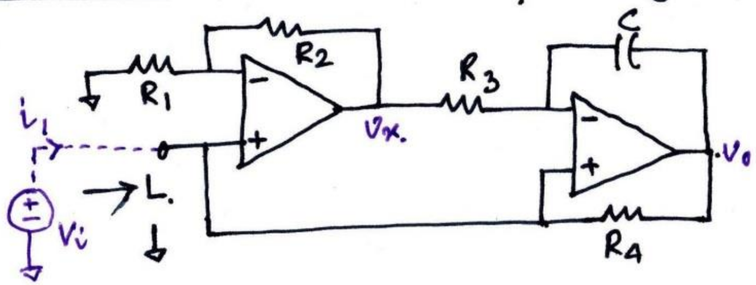
Assume $\boxed{C_2 = 1\,nF.}$

or, $R_x = \sqrt{\frac{L}{C_2}}$

$$= \sqrt{\frac{2.533m}{1n}}$$

$$= 1.592\,K\Omega.$$

Problem :- Show that the following ckt simulates a ground inductance $L = R_1R_3R_4C/R_2$.



$$\left[\begin{array}{l} i_1 = \dfrac{v_i - v_o}{R_4} \\ \& \quad v_x = \left(1 + \dfrac{R_2}{R_1}\right) v_i \quad \text{— (1)} \\ v_i - v_o = i_1 R_4 \quad \text{— (2)} \end{array}\right.$$

$$\frac{v_x - v_i}{R_3} = sC\,(v_i - v_o)$$

or, $$\left(1 + \frac{R_2}{R_1}\right)v_i - v_i = R_3 sC\left\{ v_i - v_o \right\}$$

or, $$\frac{R_2 v_i}{R_1} = R_3 sC\, v_i - R_3 sC\, v_o.$$

or, $$R_3 sC v_o = R_3 sC\, v_i - \frac{R_2}{R_1} v_i$$

or, $$(v_i - i_1 R_4) R_3 sC = R_3 sC\, v_i - \frac{R_2}{R_1} v_i \qquad [\text{using (2)}].$$

or, $$+ i_1 R_4 R_3 sC = + \frac{R_2}{R_1} v_i$$

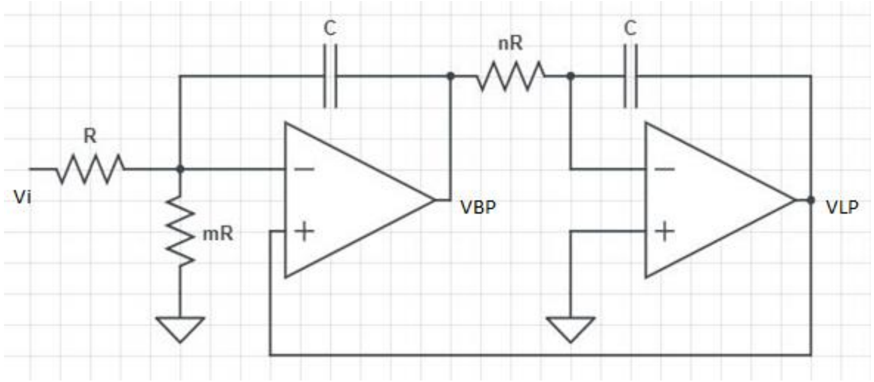
or, $$\frac{v_i}{i_1} = + \frac{s R_1 R_4 R_3 C}{R_2}.$$

or, $$Z_{in} = s\,\frac{R_1 R_4 R_3 C}{R_2}$$

$$L = \frac{R_1 R_4 R_3 C}{R_2}.$$

Try Yourself!

1. The simplified state variable filter shown in figure provides the low pass and band pass response using only two op-amps. Derive the overall transfer function V_{BP}/Vi and V_{LP}/Vi. Prove that Q= $\sqrt{(n(1+1/m))}$ and ω_o= Q/nRC.



2. Design a second order KRC low pass filter with equal component design. Find out the component values to achieve f_o= 10 kHz and Q=5. Find out the DC gain.

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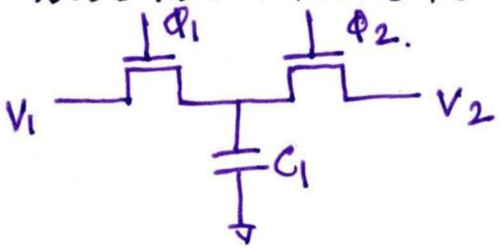
Switched Capacitor Filter

- Issues of continuous time filter:- Filter parameters are sensitive to parameter variations.
- Key features of the switched capacitor filter:-
  - a) Key elements used: switches and capacitor.
  - b) Operates as a discrete time signal processor (without using A/D or D/A converter)
  - c) Filter co-efficients are determined by capacitance ratio, which can be controlled precisely in IC design.
  - d) Provides an accurate frequency response.
  - e) Provides good linearity.
  - f) Provides good dynamic range.
  - g) Analysis is done using z-transform.
  - h) Very popular in IC design.

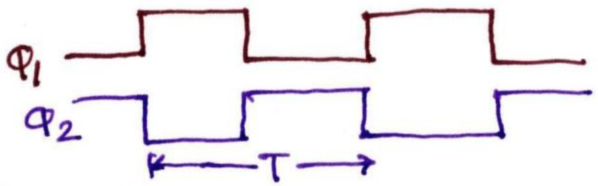
Basic operation of the switched capacitor ckt :-



Basic formula $Q = CV$ and charged conservation are used.

At ϕ_1 phase : $Q_1 = C_1 V_1$.

At ϕ_2 phase : $Q_2 = C_1 V_2$



Charge transfer over one clock period ; $\Delta Q = C_1(V_1 - V_2)$

Charge transfer is repeated in every clock period T.

If I_{avg} is the average o/p current, $I_{avg} \cdot T = C_1(V_1 - V_2)$

or, $I_{avg} = f_s C_1 (V_1 - V_2)$ where $f_s = \frac{1}{T}$.

or, $\frac{V_1 - V_2}{I_{avg}} = \frac{1}{f_s C_1}$

or, $\boxed{R_{eq} = \frac{1}{f_s C_1}}$

Example :-
$f_s = 100\,\text{kHz}$, $C_1 = 1\,\text{PF}$
$R_{eq} = \frac{1}{100K \times 1P} = 10\,\text{M}\Omega$

- When $f_s \uparrow$, same charge transfer occurs at a faster rate, $R_{eq} \downarrow$.
- If $C_1 \uparrow$, large amount of charge transfer occurs in each period, $I_{avg} \uparrow$, $R_{eq} \downarrow$.

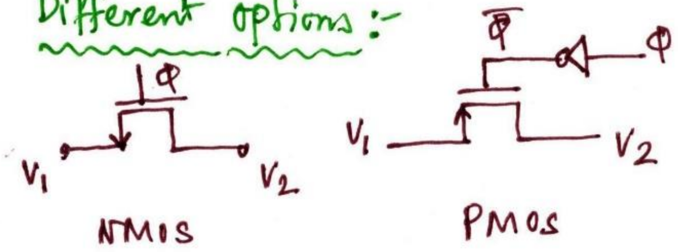
Important to note :- ✓ Resistor approximation assumes the charge transfer per cycle is constant over many cycles. Mimics low frequency behavior.
✓ For moderate frequency, discrete time analysis is required.

Different elements of switched capacitor circuits :-

a) Switch :-

- ✓ Very high resistance in off-state.
- ✓ Very low resistance in on-state, to provide small time constant & high speed operation.
- ✓ No dc offset, otherwise accuracy will be degraded.

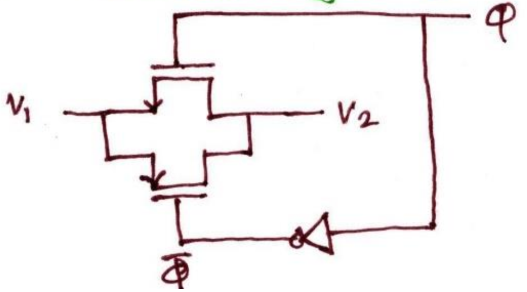
Different options :-



* NMOS introduces offset when $V_1 = V_{DD}$

* PMOS introduces offset when $V_1 = 0$

A transmission gate :-

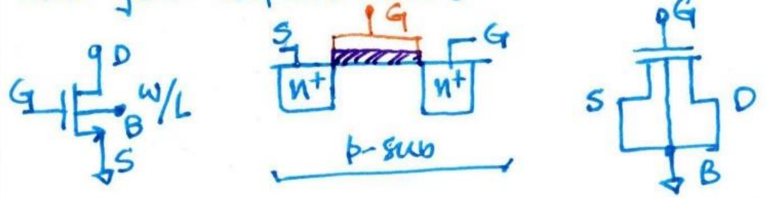


b) Capacitor :-

Different capacitor options are available in IC.

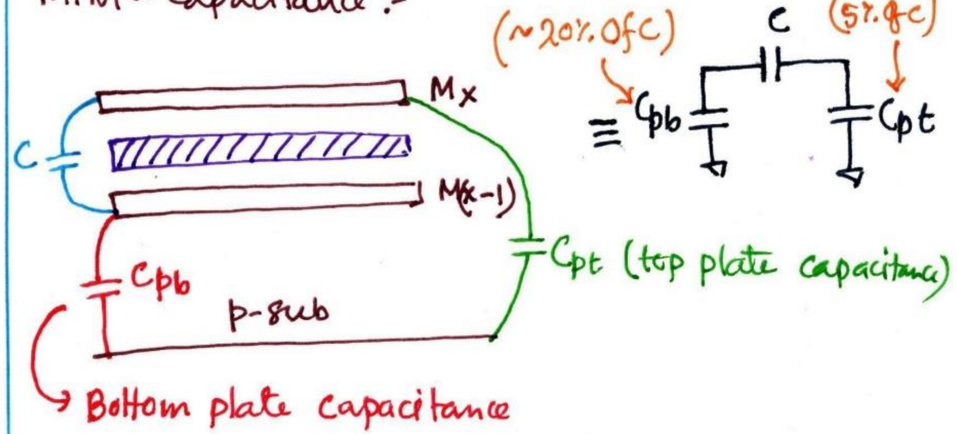
- ✓ MOS gate capacitance
- ✓ MIM-Cap : Metal-Insulator-metal.

MOS gate capacitance :-

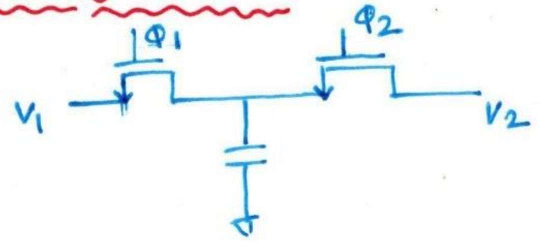


$C_g = C_{ox} WL$.

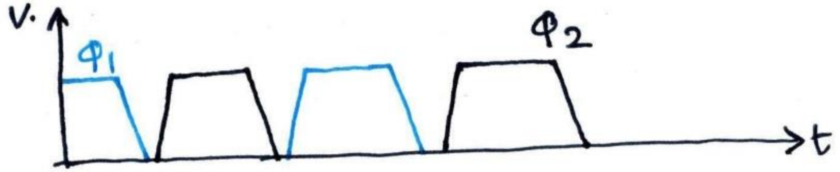
MIM-Capacitance :-

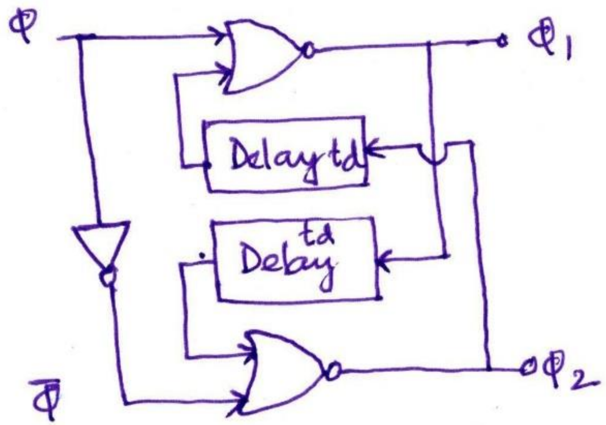


c) Non-overlapping clock generator :-



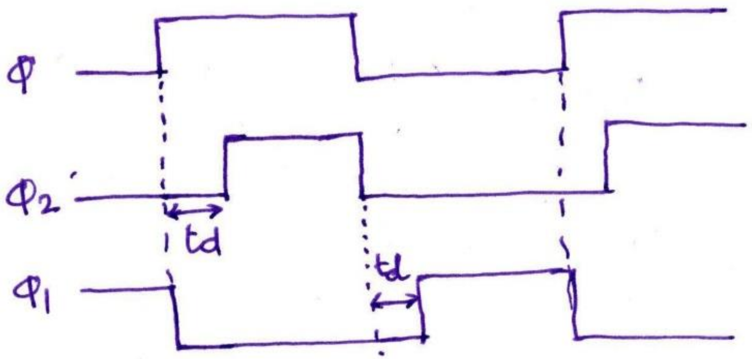
- ϕ_1 and ϕ_2 should be non-overlapping clock to gurantee no charge loss.
- Principle used : "Break before Make".
- ϕ_1 and ϕ_2 clock should have same frequency and complementary.
- Non-overlapping means they never be high at same time.





NOR Gate

1	0	0
0	1	0
1	1	0
0	0	1



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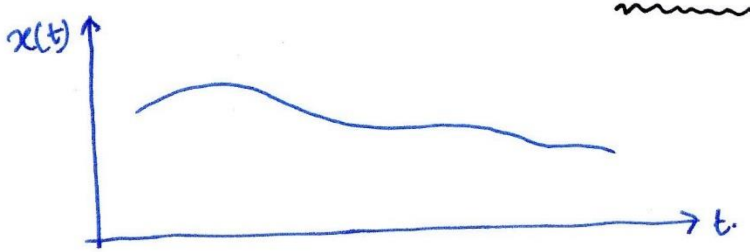
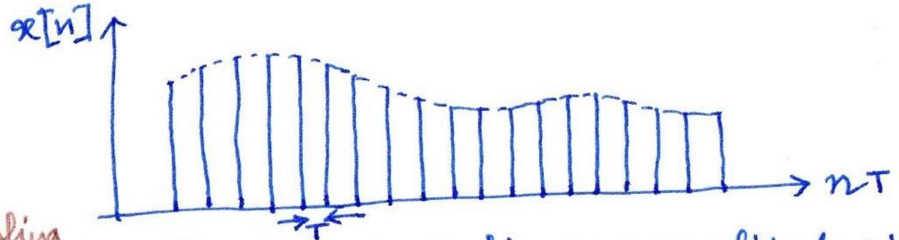
Z-transform





Continuous time, continuous amplitude signal. $x(t)$ $\xrightarrow[T.]{\text{Sampling}}$ Discrete time, continuous amplitude signal. $x[n]$

Laplace transform

$$X(s) = \int_0^{\infty} e^{-st} x(t)\, dt.$$

z- transform.

- Fourier transform of a sequence $x[n]$:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

- Z- transform of a sequence $x[n]$:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \quad \text{where } z = e^{j\omega}.$$

$x[n] \longleftrightarrow X(z)$

$x[n-1] \longleftrightarrow \ ?$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}.$$

Lets assume $m = n-1$

$n = m+1$

if $n \to -\infty$, $m \to -\infty$

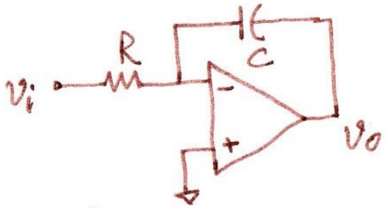
$n \to +\infty$, $m \to +\infty$

$$x[n-1] \longleftrightarrow \sum_{n=-\infty}^{\infty} x[n-1] z^{-n}. = \sum_{m=-\infty}^{\infty} x[m] z^{-(m+1)}$$

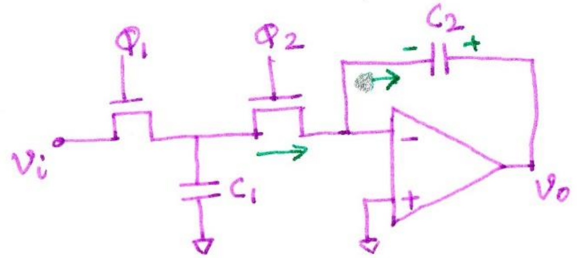
$$= z^{-1} \sum_{m=-\infty}^{\infty} x[m] z^{-m}$$

$$x[n-1] \leftrightarrow z^{-1} X(z)$$

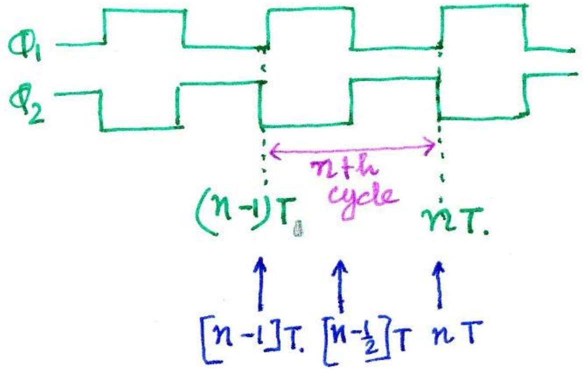
An integrator Circuit :-



Continuous time integrator



Switched capacitor integrator.



At Φ_1 phase : Charge stored in $C_1 = v_i[n-1]\,C_1$.

Charge stored in $C_2 = v_o[n-1]\,C_2$.

At Φ_2 phase : C_1 and C_2 both are loosing charges.

Final charge stored in $C_2 = v_o[n-\frac{1}{2}]\,C_2$.

Charge transferred $= v_o[n-1]\,C_2 - v_o[n-\frac{1}{2}]\,C_2$.

Charge conservation: $v_o[n-1]\,C_2 - v_o[n-\frac{1}{2}]\,C_2 = C_1\,v_i[n-1]$.

or, $v_o[n-\frac{1}{2}]\,C_2 = v_o[n-1]\,C_2 - C_1\,v_i[n-1]$.

or, $v_o[n]\,C_2 = v_o[n-1]\,C_2 - v_i[n-1]\,C_1$ As, $v_o[n-\frac{1}{2}] = v_o[n]$.

or, $v_o[n] = v_o[n-1] - \frac{C_1}{C_2} v_i[n-1]$.

Taking Z-transform :- $v_o[z] = z^{-1}v_o[z] - \frac{C_1}{C_2} z^{-1} v_i[z]$.

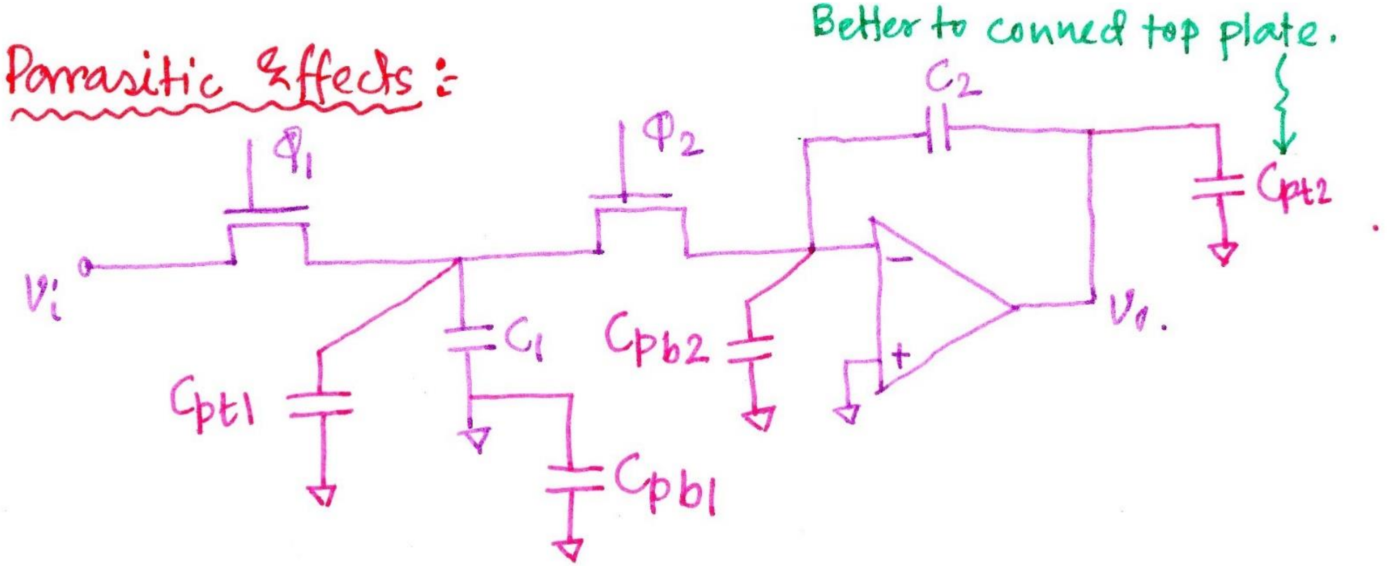
or, $$\frac{v_o[z]}{v_i[z]} = -\frac{C_1}{C_2}\,\frac{z^{-1}}{1-z^{-1}} = -\frac{C_1}{C_2}\,\frac{1}{(z-1)}$$

$z^{-1} \rightarrow$ delay

Inverting

If matching is perfect, it comes as ratioed form.

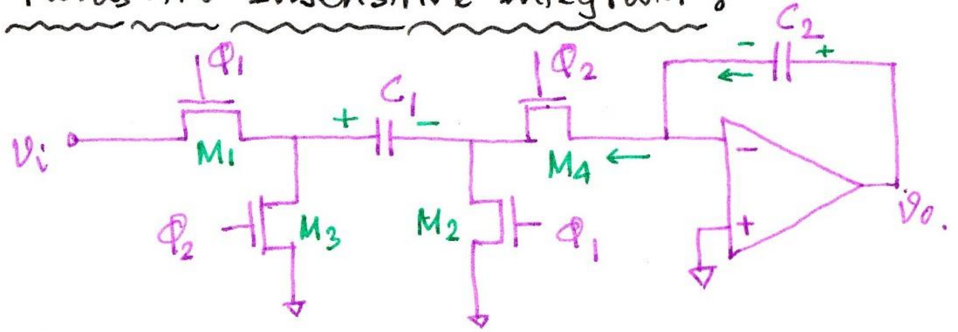
Parasitic Effects :-



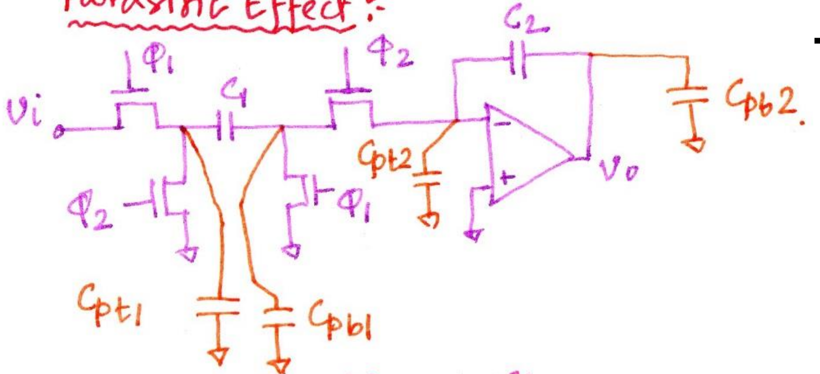
- ✓ C_{pb1} and C_{pb2} does not have any effect.
- ✗ C_{pt1} comes in parallel to C_1.
- C_{pt2} acts as an output parasitic/load capacitance. It limits the speed of response. However, it does not change final settling value.

Modified transfer function : $$\frac{V_o[z]}{V_i[z]} = -\frac{(C_1 + C_{pt1})}{C_2} \cdot \frac{1}{z-1}.$$ Parasitic sensitive.

Parasitic Insensitive Integrator :-



Parasitic Effect :-



At ϕ_1 phase : charge stored at $C_1 = V_i[n-1]C_1$.

Charge stored at $C_2 = V_o[n-1]C_2$.

At ϕ_2 phase :- C_1 looses charge and C_2 gain charge.

Final charge at $C_2 = V_o[n-\frac{1}{2}]C_2 = V_o[n]C_2$

Charge transferred $= V_o[n]C_2 - V_o[n-1]C_2$

Charge conservation :- $V_o[n]C_2 - V_o[n-1]C_2 = V_i[n-1]C_1$

or, $V_o[n]C_2 = V_o[n-1]C_2 + V_i[n-1]C_1$

Taking z-transform : $V_o[z]C_2 = z^{-1}V_o[z]C_2 + z^{-1}V_i[z]C_1$

or, $V_o[z]\left[1-z^{-1}\right]C_2 = z^{-1}V_i[z]C_1$

or, $$\frac{V_o[z]}{V_i[z]} = \frac{z^{-1}C_1}{C_2(1-z^{-1})} = \frac{C_1}{C_2}\frac{z^{-1}}{1-z^{-1}} = \frac{C_1}{C_2(z-1)}.$$

(noninverting) $z^{-1} \rightarrow$ delay

At ϕ_1 : $\left\{\begin{array}{l} C_{pt1} \rightarrow V_i \\ C_{pb1} \rightarrow \text{grounded} \end{array}\right.$

At ϕ_2 : $\left\{\begin{array}{l} C_{pt1} \rightarrow \text{grounded} \\ C_{pb1} \rightarrow \text{grounded} \end{array}\right.$

no effect

$C_{pt2} \rightarrow$ grounded

$C_{pb2} \rightarrow$ provides delay.

It gives parasitic insensitive tr. fn.

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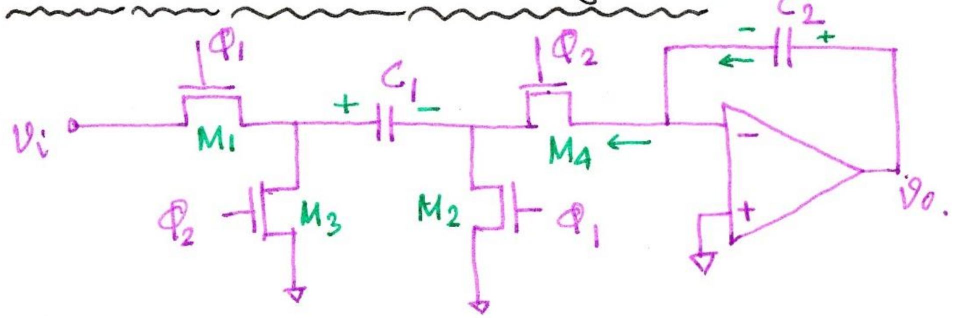
Parasitic Insensitive Integrator :-



At ϕ_1 phase : charge stored at $C_1 = V_i[n-1]C_1$.

Charge stored at $C_2 = V_o[n-1]C_2$.

At ϕ_2 phase :- C_1 looses charge and C_2 gain charge.

Final charge at $C_2 = V_o[n-\frac{1}{2}]C_2 = V_o[n]C_2$

Charge transferred $= V_o[n]C_2 - V_o[n-1]C_2$

Charge conservation :- $V_o[n]C_2 - V_o[n-1]C_2 = V_i[n-1]C_1$

or, $V_o[n]C_2 = V_o[n-1]C_2 + V_i[n-1]C_1$

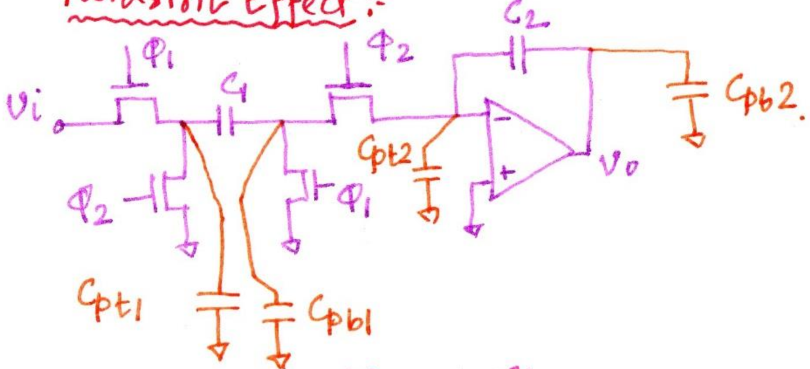
Taking z-transform : $V_o[z]C_2 = z^{-1}V_o[z]C_2 + z^{-1}V_i[z]C_1$

or, $V_o[z][1-z^{-1}]C_2 = z^{-1}V_i[z]C_1$

or, $$\frac{V_o[z]}{V_i[z]} = \frac{z^{-1}C_1}{C_2(1-z^{-1})} = \frac{C_1}{C_2}\frac{z^{-1}}{1-z^{-1}} = \frac{C_1}{C_2(z-1)}.$$

(noninverting) $z^{-1} \rightarrow$ delay

Parasitic Effect :-



At ϕ_1 : $C_{pt1} \rightarrow v_i$, $C_{pb1} \rightarrow$ grounded

At ϕ_2 : $C_{pt1} \rightarrow$ grounded, $C_{pb1} \rightarrow$ grounded

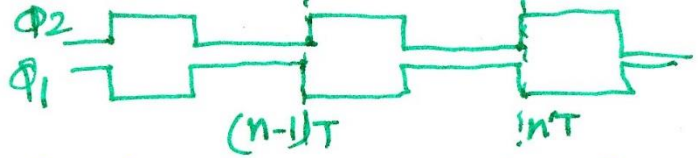
no effect

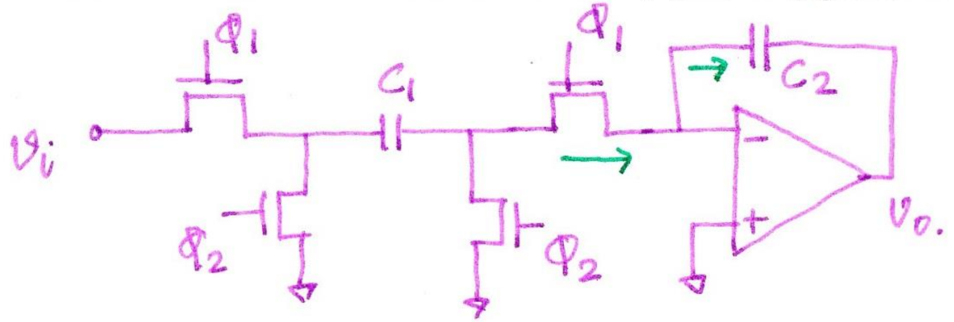
$C_{pt2} \rightarrow$ grounded

$C_{pb2} \rightarrow$ provides delay.

It gives parasitic insensitive tr. fn.

A delay free, parasitic insensitive, inverting integrator:-





At Φ_2 phase: Charge stored at $C_2 = C_2 v_o[n-1]$.

charge stored at $C_1 = 0$.

At Φ_1 phase: Charge stored at $C_1 = v_i[n] C_1$.

· charge stored at $C_2 = v_o[n] C_2$.

C_2 looses charge

Charge conservation: Total charge at Φ_1 and Φ_2 phases are same.

$$v_i[n] C_1 + v_o[n] C_2 = C_2 v_o[n-1].$$

or, $$+v_o[n] C_2 - C_2 v_o[n-1] = -v_i[n] C_1$$

or, $$v_o[n] - v_o[n-1] = -\frac{C_1}{C_2} v_i[n].$$

or, $$v_o[z] - z^{-1} v_o[z] = -\frac{C_1}{C_2} v_i[z].$$

or, $$v_o[z]\left[1 - z^{-1}\right] = -\frac{C_1}{C_2} V_i[z]$$

or, $$\frac{V_o[z]}{V_i[z]} = -\frac{C_1/C_2}{1-z^{-1}} = -\frac{C_1}{C_2}\frac{z}{(z-1)}$$

Delay free, inverting, parasitic insensitive.

How can we approximate the integrator transfer function as an ideal continuous time integrator?

Transfer function of an integrator.

$$H(z) = -\frac{C_1}{C_2}\frac{z^{-1}}{1-z^{-1}} = -\frac{C_1}{C_2}\frac{z^{-1/2}}{z^{+1/2}-z^{-1/2}}$$

$z = e^{sT} = e^{j\omega T} = \cos(\omega T) + j\sin(\omega T)$ where T = sampling period. $= 1/f_s$

ω = input signal frequency.

$$z^{1/2} = e^{j\omega T/2} = \cos\left(\frac{\omega T}{2}\right) + j\sin\left(\frac{\omega T}{2}\right)$$

$$z^{-1/2} = e^{-j\omega T/2} = \cos\left(\frac{\omega T}{2}\right) - j\sin\left(\frac{\omega T}{2}\right).$$

$$H(z) = -\frac{C_1}{C_2}\frac{z^{-1/2}}{2j\sin\frac{\omega T}{2}}.$$

To get an integral action, $\omega << 1/T$. or, $\omega T << 1$, then the switched capacitor ckt will act as a resistor.

$$H(z) \approx -\frac{C_1}{C_2}\frac{z^{-1/2}}{2j\frac{\omega T}{2}} = -\frac{C_1}{C_2}\frac{z^{-1/2}}{j\omega T}.$$

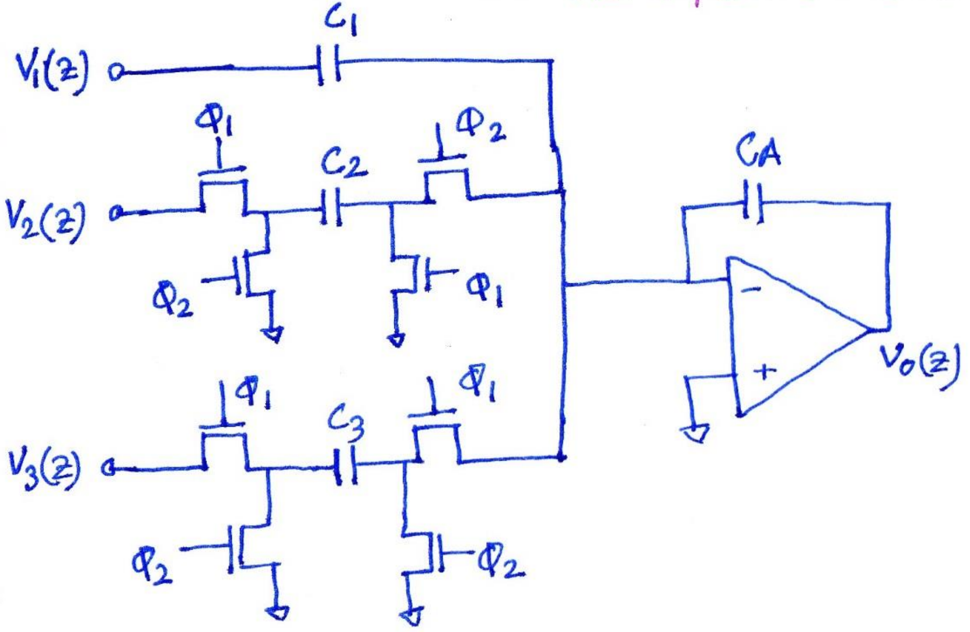
$z^{-1/2}$ is just a delay term; it has nothing to do with integral action.

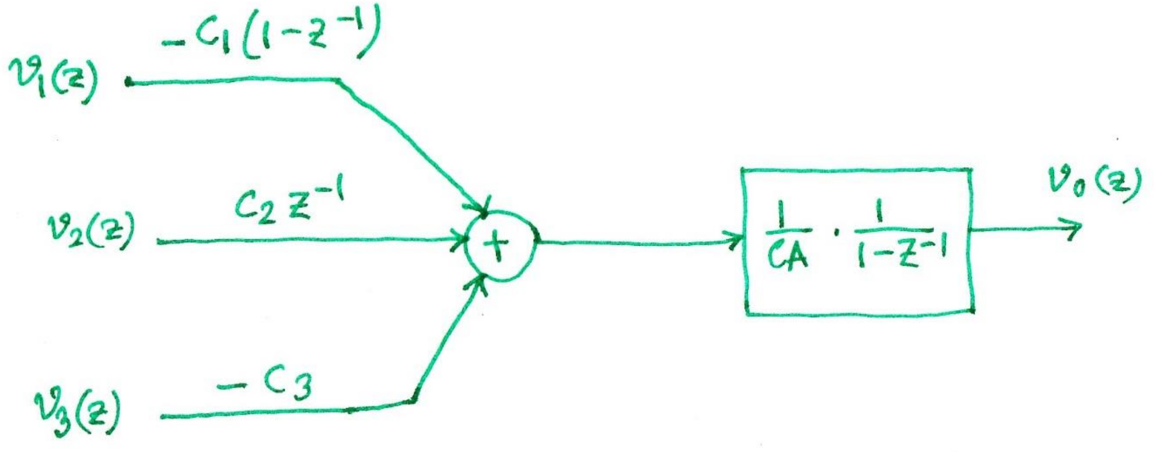
Integrator gain $K_1 = \frac{C_1}{C_2 T}$

In continuous time $\rightarrow \frac{1}{s}$. In discrete time $\rightarrow \frac{1}{1-z^{-1}}$

Signal flow graph analysis :-

- Charge transfer equation for larger circuit may be tedious.
- Few rules can be developed to analyse the circuit graphically. Similar to block diagram.





$$\frac{V_{o1}(z)}{V_1(z)} = -\frac{C_1}{C_A}$$

$$\frac{V_{o2}(z)}{V_2(z)} = \frac{C_2}{C_A}\cdot\frac{z^{-1}}{1-z^{-1}}$$

$$\frac{V_{o3}(z)}{V_3(z)} = -\frac{C_3}{C_A}\cdot\frac{1}{1-z^{-1}}$$

Applying voltage superposition;-

$$V_o(z) = -\frac{C_1}{C_A}V_1(z) + \frac{C_2}{C_A}\cdot\frac{z^{-1}}{1-z^{-1}}V_2(z) - \frac{C_3}{C_A}\cdot\frac{1}{1-z^{-1}}V_3(z)$$

EE60032: Analog Signal Processing

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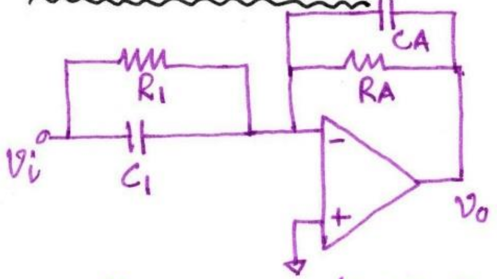
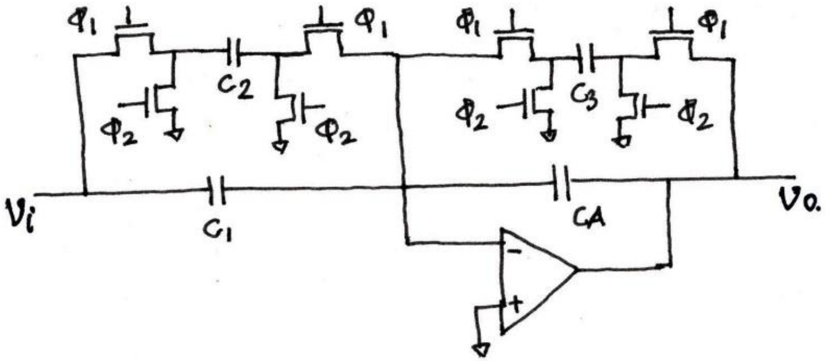
First Order Filter :-

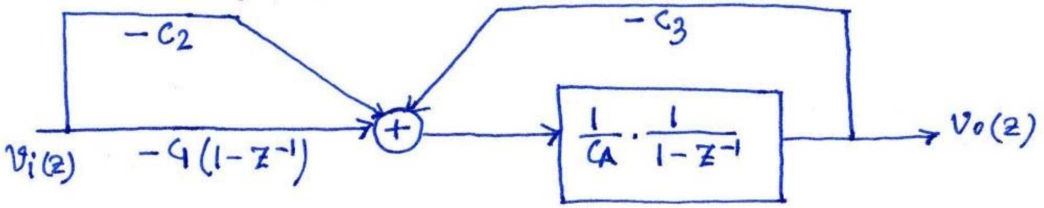


$$\frac{V_o}{V_i} = -\frac{R_A}{R_1}\cdot\frac{(1+sR_1C_1)}{(1+sR_AC_A)}$$

First order, continuous time, Active RC filter.



First order, switched capacitor, delay free, active RC filter.



$$-\frac{C_1(1-z^{-1})V_i(z)}{C_A(1-z^{-1})} - \frac{C_2V_i(z)}{C_A(1-z^{-1})} - \frac{C_3V_o(z)}{C_A(1-z^{-1})} = V_o(z)$$

or, $-C_1(1-z^{-1})V_i(z) - C_2V_i(z) = C_A(1-z^{-1})V_o(z) + C_3V_o(z)$

$$H(z) = \frac{V_o(z)}{V_i(z)} = -\frac{C_1(1-z^{-1}) + C_2}{C_3 + C_A(1-z^{-1})}$$

$$= -\frac{\left(\frac{C_1+C_2}{C_A}\right)z - \frac{C_1}{C_A}}{\left(1+\frac{C_3}{C_A}\right)z - 1}$$

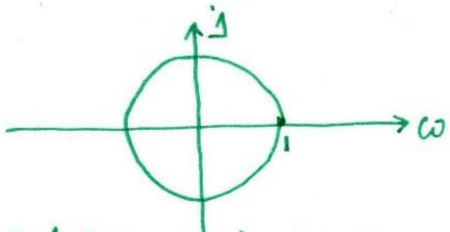
Pole $Z_p = \frac{C_A}{C_A + C_3} < 1$

zero $Z_z = \frac{C_1}{C_1 + C_2} < 1$

As, $z = e^{j\omega T}$, $\omega \to 0$, $z \to 1$

So, DC gain can be found by setting $z = 1$.

$$H(1) = -\frac{C_2}{C_3}$$



Poles and zero are inside the unit circle, hence always stable.

How to get pole and zero locations under $\omega T \ll 1$.

$$H(z) = \frac{V_o(z)}{V_i(z)} = -\frac{\left(\frac{C_1+C_2}{C_A}\right)z - \frac{C_1}{C_A}}{\left(1+\frac{C_3}{C_A}\right)z - 1} = -\frac{\frac{C_1}{C_A}(z-1) + \frac{C_2}{C_A}z}{z-1+\frac{C_3}{C_A}z} = -\frac{\frac{C_1}{C_A}\left(z^{1/2}-z^{-1/2}\right) + \frac{C_2}{C_A}z^{1/2}}{z^{1/2}-z^{-1/2}+\frac{C_3}{C_A}z^{+1/2}}$$

$$H(j\omega T) = \frac{V_o(e^{j\omega T})}{V_i(e^{j\omega T})} = -\frac{\frac{C_1}{C_A}\,2j\sin\frac{\omega T}{2} + \frac{C_2}{C_A}\left(\cos\frac{\omega T}{2} + j\sin\frac{\omega T}{2}\right)}{2j\sin\frac{\omega T}{2} + \frac{C_3}{C_A}\left(\cos\frac{\omega T}{2} + j\sin\frac{\omega T}{2}\right)}$$

$$= -\frac{\frac{2C_1+C_2}{C_A}\,j\sin\left(\frac{\omega T}{2}\right) + \frac{C_2}{C_A}\cos\frac{\omega T}{2}}{\left(2+\frac{C_3}{C_A}\right)j\sin\left(\frac{\omega T}{2}\right) + \frac{C_3}{C_A}\cos\frac{\omega T}{2}}$$

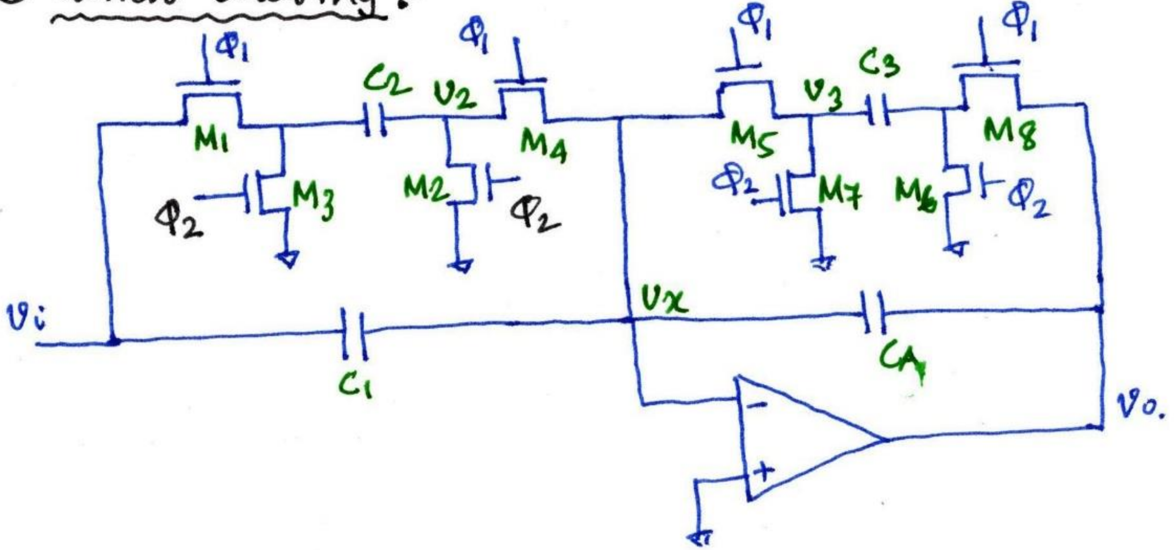
when $\omega T \ll 1$, input signal is changing very slowly compared to the sampling freq.

$$\approx -\frac{\frac{2C_1+C_2}{C_A}\,j\frac{\omega T}{2} + \frac{C_2}{C_A}}{\left(2+\frac{C_3}{C_A}\right)j\frac{\omega T}{2} + \frac{C_3}{C_A}}$$

Zero, $\omega_z T = \dfrac{2C_2/C_A}{\frac{2C_1+C_2}{C_A}} = \dfrac{C_2}{\left(1+\frac{C_2}{2C_1}\right)}$

Pole, $\omega_p T = \dfrac{C_3/C_A}{\left(1+C_3/2C_A\right)}$

Switch sharing :-



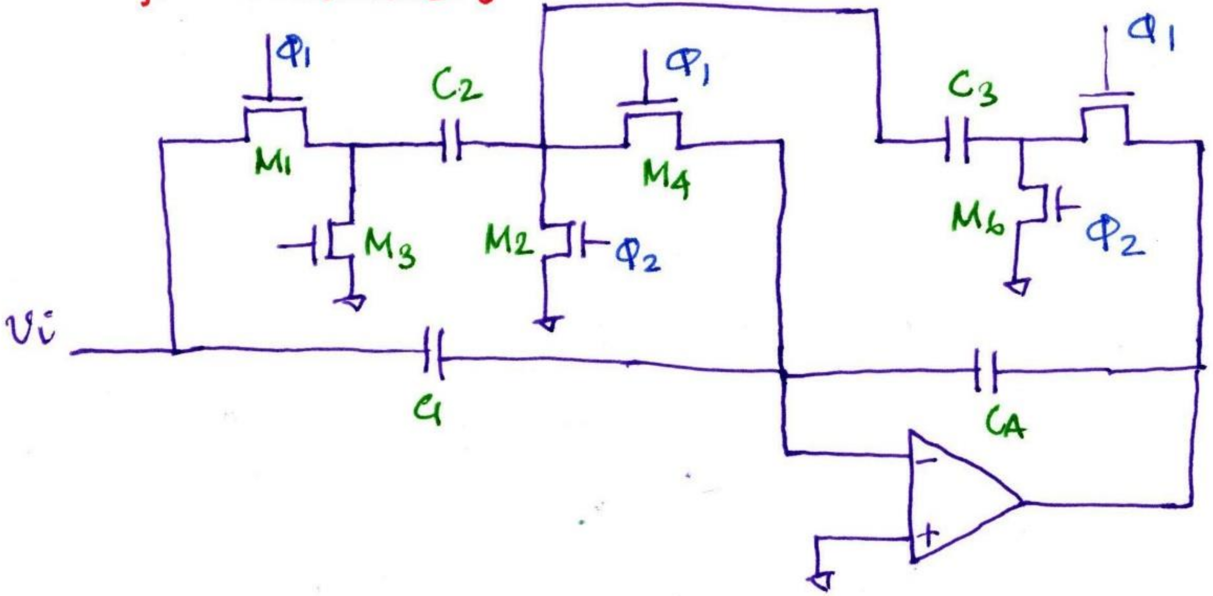
At Φ_1 phase: $V_2 = V_3$

At Φ_2 phase: $V_2 = V_3 = 0$

So, V_2 and V_3 can be shorted.

However, V_x is floating, $V_x \neq (V_2, V_1)$ at Φ_2.

Modified structure :-



M_7 & M_5 are removed.

Problem: The first order filter as shown in the previous slide, find the value of C_2 needed for a first order low pass filter, that has $C_1 = 0$ and a pole at $\frac{1}{64}$ th of the sampling frequency using approximate equation. The low frequency gain should be 1.

Generalised expression: $H(z) = -\dfrac{\left(\frac{C_1+C_2}{C_A}\right)z - \frac{C_1}{C_A}}{\left(1+\frac{C_3}{C_A}\right)z - 1}$

DC gain $H(1) = -\dfrac{\frac{C_1+C_2}{C_A} - \frac{C_1}{C_A}}{\left(1+\frac{C_3}{C_A}\right)-1} = -\dfrac{C_2/C_A}{C_3/C_A}$ if $C_1 = 0$.

$= -C_2/C_3 = -1$ if $C_2 = C_3$.

$$\omega_p T = +\frac{C_3/C_A}{\left(1+C_3/2C_A\right)}$$

As $f_p = \dfrac{f_s}{64} = \dfrac{1}{64T}$.

or, $\dfrac{\omega_p}{2\pi} = \dfrac{1}{64T}$.

or, $\omega_p T = \dfrac{2\pi}{64}$

$\dfrac{2\pi}{64} = \dfrac{C_3/C_A}{1+C_3/2C_A}$

or, $\dfrac{2\pi}{64} + \dfrac{2\pi}{64}\cdot\dfrac{C_3}{2C_A} = \dfrac{C_3}{C_A}$.

or, $\dfrac{C_3}{C_A}\left[1-\dfrac{2\pi}{128}\right] = \dfrac{2\pi}{64}$.

or, $\dfrac{C_3}{C_A} = \left[\dfrac{2\pi/64}{1-2\pi/128}\right] = 0.1032$

If $C_A = 10\,pF$, $C_3 = 1.032\,pF$, $C_2 = 1.032\,pF$.